Internet Appendix to "Intermediated Investment Management"

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This note presents the proofs of several propositions in "Intermediated Investment Management."

I. Proof of Proposition 4 and Corollary 1

Proof. Equation (24) in the paper shows that the portfolio manager's profit is maximized either at $\delta = c_A/\eta$ or $\delta = \bar{\delta}$ in the case $\eta > 1$. Consider first the scenario $\delta = c_A/\eta$, in which the active fund and the passive fund have the same expected return. From equation (24) we see Π_P is maximized at

$$A_I = \frac{1}{2\gamma}[\alpha - R_m - c_A + (\eta - 1)\delta] - A_D = \frac{1}{2\gamma}[\alpha - R_m - c_A/\eta] - A_D,$$

where $A_D = \frac{kA_m^k c_A^{k-1}}{(k-1)(C_0 R_m)^{k-1}}$. Substituting this result and $\delta = c_A/\eta$ into the second case of equation (18) in the paper yields the optimal management fee f_P^* stated in the proposition.

In the second scenario $\delta = \bar{\delta}$, the active fund underperforms the passive fund, and $A_D = 0$. Π_P is maximized at

$$A_I' = \frac{1}{2\gamma}[\alpha - R_m - c_A + (\eta - 1)\bar{\delta}].$$

Substituting this back into the second case of equation (18) and noting that $A_D = 0$ and $\delta = \bar{\delta}$, we have the optimal management fee, $f_P^{*'}$, for this scenario.

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To prove Corollary 1, note that the portfolio manager's profit in the first scenario is

$$\Pi_P = \frac{(\alpha - R_m - c_A/\eta)^2}{4\gamma} + \frac{A_D c_A}{\eta},$$

and in the second scenario it is

$$\Pi_P' = \frac{[\alpha - R_m - c_A + (\eta - 1)\bar{\delta}]^2}{4\gamma}.$$

The difference between the portfolio manager's profits under these two scenarios is then

$$\begin{aligned}
\Delta\Pi_P &= \Pi_P - \Pi_P' \\
&= \frac{c_A^2/\eta^2 - [c_A - (\eta - 1)\bar{\delta}]^2 - 2(\alpha - R_m)[c_A/\eta - c_A + (\eta - 1)\bar{\delta}]}{4\gamma} + A_D c_A/\eta.
\end{aligned}$$

If $\Delta\Pi_P > 0$, the portfolio manager chooses the first (equal performance) equilibrium with $\delta = c_A/\eta$. Otherwise she chooses the second (underperformance) equilibrium with $\delta = \bar{\delta}$. To see which equilibrium is more likely to occur, we take the partial derivative of $\Delta\Pi_P$ with respect to various model parameters. A negative partial derivative means the second equilibrium is more likely to occur as the parameter value increases.

First, note that

$$\frac{\partial \Delta\Pi_P}{\partial \bar{\delta}} = -\frac{1}{2\gamma}[\alpha - R_m - c_A + (\eta - 1)\bar{\delta}](\eta - 1) < 0,$$

where the inequality follows from our assumptions $\alpha > R_m + c_A$, $\eta > 1$, and $\bar{\delta} > c_A/\eta$. Therefore, when $\bar{\delta}$ is high, it is more likely that the portfolio manager prefers the underperformance equilibrium.

Second, note that C_0 and k affect $\Delta\Pi_P$ only through A_D. From the expression of A_D, it is easy to see that A_D is decreasing in both C_0 and k:

$$\frac{\partial log(A_D)}{\partial C_0} = -(k-1)/C_0 < 0,$$

$$\frac{\partial log(A_D)}{\partial k} = (\frac{1}{k} - \frac{1}{k-1}) + log(\frac{A_m c_A}{C_0 R_m}) = (\frac{1}{k} - \frac{1}{k-1}) + log(\frac{A_m}{A^*}) < 0.$$

Since $\Delta\Pi_P$ increases in A_D, it follows that $\Delta\Pi_P$ decreases in both C_0 and k.

Third, note that

$$\frac{\partial \Delta\Pi_P}{\partial \alpha} = -\frac{1}{2\gamma}[c_A/\eta - c_A + (\eta-1)\bar{\delta}] < 0,$$

where the inequality follows from our assumptions $\eta > 1$ and $\bar{\delta} > c_A/\eta$.

Finally, we have

$$\begin{aligned}
\frac{\partial \Delta\Pi_P}{\partial \eta} &= \frac{1}{2\gamma}\{(\alpha - R_m - c_A/\eta)c_A/\eta^2 - [\alpha - R_m - c_A + (\eta-1)\bar{\delta}]\bar{\delta}\} - A_D c_A/\eta^2 \\
&< \frac{1}{2\gamma}\{(\alpha - R_m - c_A/\eta)\bar{\delta} - [\alpha - R_m - c_A + (\eta-1)\bar{\delta}]\bar{\delta}\} - A_D c_A/\eta^2 \\
&= \frac{1}{2\gamma}[(-c_A/\eta + c_A - (\eta-1)\bar{\delta})]\bar{\delta} - A_D c_A/\eta^2 \\
&< -A_D c_A/\eta^2 \\
&< 0,
\end{aligned}$$

where the first inequality follows from the assumptions $\alpha > R_m + c_A$, $\eta > 1$, and $\bar{\delta} > c_A/\eta$, and the second inequality follows from the assumptions $\eta > 1$ and $\bar{\delta} > c_A/\eta$.

II. Proof of Proposition 7

Proof. Combining the two constraints in problem (34) in the paper we immediately obtain equation (36). Substituting this expression back into the objective function and differentiating, we have

$$\begin{aligned} \frac{\partial \Pi_P}{\partial A_D} &= \alpha - R_m - 2\gamma A_D - \frac{\lambda k}{k-1} A_D^{1/(k-1)}, \\ \frac{\partial^2 \Pi_P}{\partial A_D^2} &= -2\gamma - \frac{\lambda k}{(k-1)^2} A_D^{(2-k)/(k-1)} < 0. \end{aligned}$$

Equation (35) in the paper is obtained by setting the first-order condition above equal to zero. Since Π_P is strictly concave when $A_D > 0$, the first-order condition is both a necessary and sufficient condition for the solution to this maximization problem; furthermore, the optimal A_D is unique. To prove the existence of an interior solution, $0 < A_D < W - C_0$, to the first-order condition, note that $\frac{\partial \Pi_P}{\partial A_D} > 0$ if $A_D = 0$. Due to the monotonicity of the first derivative, it suffices to show this derivative becomes negative as $A_D \to W - C_0$, that is, as A_D converges to the aggregate wealth of the economy net of the search cost C_0. This is guaranteed by condition (6) in the paper.

III. Proof of Proposition 8

Proof. In the case without financial advisers, the number of direct investors is the same as the number of investors investing in the active portfolio. Denote the total surplus of the (direct) investors, relative to the default of passive investment, by S^0. We have

$$\begin{aligned} S^0 &= \int_{A_0^*}^{+\infty} [x(\alpha - \gamma A_D^0)(1 - f_P) - (x + C_0) R_m] f(x) dx \\ &= [(\alpha - \gamma A_D^0)(1 - f_P) - R_m] A_D^0 - \theta^0 C_0 R_m, \end{aligned}$$

where A_0^* is the threshold level of wealth (net of C_0) that makes the marginal investor indifferent between the passive fund and the active portfolio, $A_D^0 = \int_{A_0^*}^{+\infty} x f(x) dx = \frac{k A_m^k}{(k-1)(A_0^*)^{k-1}}$, $\theta^0 \equiv \int_{A_0^*}^{+\infty} f(x) dx = (\frac{A_m}{A_0^*})^k$.

In the equilibrium with independent advisers, the net return of the active fund is equal to $R_m + c_A$. Therefore, the (direct) investor's surplus, S^1, is given by

$$S^1 = \int_{\frac{C_0 R_m}{c_A}}^{+\infty} [x(R_m + c_A) - (x + C_0)R_m)]f(x)dx$$
$$= A_D^1 c_A - \theta^1 C_0 R_m,$$

where $A_D^1 = \frac{k A_m^k c_A^{k-1}}{(k-1)(C_0 R_m)^{k-1}}$, and $\theta^1 \equiv (\frac{c_A A_m}{C_0 R_m})^k$.

Similarly, since the net return of the active portfolio in the case with subsidized advisers equals $R_m + c_A - \delta$, the total surplus of the (direct) investors in the subsidized adviser equilibrium is given by

$$S^2 = A_D^2 (c_A - \delta) - \theta^2 C_0 R_m,$$

where $A_D^2 = \frac{k A_m^k (c_A - \delta)^{k-1}}{(k-1)(C_0 R_m)^{k-1}}$, and $\theta^2 \equiv (\frac{(c_A - \delta) A_m}{C_0 R_m})^k$.

In the unsophisticated investor case, high net worth investors have a deadweight loss of C_0. The fraction of investors who pay this cost is the same as in the case without rebate, that is, θ^1. The indirect investors earn an expected return that is $\eta\delta$ lower than the passive return, where δ equals either c_A/η or $\bar{\delta}$. Therefore, the total investor surplus in this case is

$$S^3 = -A_I^3 * \eta\delta - \theta^1 C_0 R_m < 0.$$

Note that investor surplus S^0, S^1, and S^2 must all be strictly positive, otherwise no rational investors will pay the search cost. Therefore $S^3 < 0$ is lowest among all the four equilibria. To prove $S^1 > S^2$, note that

$$S^1 - S^2 = A_D^2 \delta + (A_D^1 - A_D^2) c_A - (\theta^1 - \theta^2) C_0 R_m = A_D^2 \delta + \int_{\frac{C_0 R_m}{c_A}}^{\frac{C_0 R_m}{c_A - \delta}} x f(x) [c_A - \frac{C_0 R_m}{x}] dx > 0.$$

This equation indicates that investors' welfare loss due to the existence of kickbacks can be decomposed into two parts: investors who remain in the direct channel lose $A_D^2\delta$, and investors who would originally choose the direct channel but are forced to switch to the indirect channel because of kickbacks lose $(A_D^1 - A_D^2)c_A - (\theta^1 - \theta^2)C_0 R_m$. Both components are strictly positive.

Adding the portfolio manager's profit to investors' surplus, we get total welfare U^0, U^1, U^2, and U^3 in Proposition 8. To prove $U^1 > U^2$, recall that allowing kickbacks increases the portfolio manager's profit by $A_D^2\delta$ (equation (23)), and thus the first component of the investor welfare loss described above is exactly offset by the gain of the portfolio manager. However, the second component is a deadweight loss.

To prove $U^1 > U^3$, we first compare the independent adviser equilibrium with the unsophisticated investor equilibrium with $\delta = c_A/\eta$. Using the expressions for investor surplus and the portfolio manager's profit for both cases, we derive

$$U^1 - U^3 = \frac{c_A^2}{4\gamma}(1 - \frac{1}{\eta^2}) + \frac{A_I^1 c_A}{\eta} > 0,$$

where A_I^1 denotes the amount of indirect investment in the independent adviser equilibrium. Similarly, comparing the independent adviser equilibrium with the unsophisticated investor equilibrium with $\delta = \bar{\delta}$, we have

$$U^1 - U^3 = A_D^1 c_A + (A_D^1 + A_I^1)\bar{\delta} + \frac{(\eta - 1)(\eta + 1)\bar{\delta}^2}{4\gamma} > 0.$$

This completes our proof of Proposition 8.