Mutual Fund Families and Performance Evaluation

David P. Brown Youchang Wu*

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Abstract

We develop a model of performance evaluation for mutual funds within a family. Good family performance has two competing effects on the estimate of a member fund's alpha-generating skill and its inflows: a positive common-skill effect, and a negative correlated-noise effect. The sensitivities of the skill estimate to fund and family performance depend on the weight of the family skill component, the correlation of noise in fund returns, the number of funds in the family, as well as fund size and fund age. Empirical estimates of mutual fund flow sensitivities show patterns consistent with rational cross-fund learning within families.

Key words: mutual fund family, fund flow, performance evaluation, learning

JEL codes: G23, G11

*Both authors are in the Department of Finance, Investment and Banking, School of Business, University of Wisconsin-Madison. Mail address: Grainger Hall, 975 University Avenue, Madison, WI 53706. Email addresses: dbrown@bus.wisc.edu, and ywu@bus.wisc.edu. Phone numbers: 608-2655281 (Brown), 608-8902595 (Wu). We thank Jonathan Berk, Michael Brennan, Pierre Collin-Dufresne, Thomas Dangl, Zhiguo He, Bryan Lim (discussant), Lubos Pastor (discussant), Matthew Spiegel (discussant), Luke Taylor, Neal Stoughton, Hong Yan (discussant), Tong Yao (discussant), Josef Zechner, seminar participants at the Utah Winter Finance Conference in 2012, American Finance Association meetings in 2012, Western Finance Association meetings in 2011, Financial Intermediation Research Society meetings in 2011, China International Conference in Finance in 2011, University of Wisconsin-Madison, University of Illinois at Urbana-Champaign, University of Illinois at Chicago, University of Technology Sydney, and Vienna Graduate School of Finance for helpful discussions.

Investment outcomes are driven by skill and by luck. A fundamental issue in delegated portfolio management is performance evaluation; that is, to distinguish skill from luck. This distinction is crucial for appropriate selection of funds and compensation of fund managers. Most methods of performance evaluation focus on the records of individual funds in isolation, apart from any relevant information contained in other funds in the same family. The objective of this paper is to provide a theoretical framework for evaluating the performance of mutual funds within families, and to examine empirically how investors incorporate both fund and family performance information when they allocate money across funds.

Most mutual funds belong to a family. Funds in the same family often share some common resources. One simple example is that multiple funds may be managed by the same fund manager or management team. Also, managers may share information, opinions and expertise with each other even when they are managing different funds. Furthermore, funds in a family often share the same team of macroeconomic, industry and security analysts, who generate investment ideas, and they typically have access to the same trading desks, legal counselors and outside experts.[1] Other examples of family resources include information systems, portfolio analysis software, risk management tools, performance measurement procedures, and fund governance mechanisms. As a result, a fund's alpha is jointly determined by fund-specific characteristics, such as the skills of its manager and supporting staff, its specific focuses and investment strategies, and by the quality of common resources.

Some aspects of family resources affect fund alphas systematically, but have little impact on funds' exposure to idiosyncratic risks. Examples are operating efficiency, which translates into high or low expense ratios; the quality of trading desks and risk management process; and the effectiveness of fund governance and manager compensation schemes. However, the use of many other family resources has the effect of increasing the correlations of both

[1] Based on an extensive survey database, Cheng, Liu, and Qian (2006) report that fund managers on average place a weight of over 70% on internal ("buy-side") analysts as a source of research-based information.

alphas and idiosyncratic shocks to fund performance, as it induces member funds to tilt their portfolios in similar directions, meaning that they over- or underweight the same securities relative to their benchmarks. For example, a single idea from the analyst pool can lead several funds to simultaneously increase or decrease positions in a security. These funds are then subject to correlated shocks to their performance.[2]

Given the considerations above, how should one evaluate the alpha-generating skill of a mutual fund in a family? More specifically, how does an estimate of the skill depend on a fund's own performance, and the performance of other funds in the family? How do the sensitivities of the estimate to fund and family performance change over time? And how do they vary with family characteristics, including the number of funds in the family, the weight of common component in the alpha-generating skill, and the correlation of noise in fund returns? How do mutual fund flows respond to fund and family performance in equilibrium if investors learn optimally? And finally, is there evidence that investors respond to fund and performance in a manner that is consistent with optimal learning?

To answer these questions, we develop a continuous-time model in which a fund's alpha is driven by a combination of a fund-specific skill and a common skill shared by all funds in the family. We refer to this combination as the composite skill, the fund-specific component as the fund skill, and the common component as the family skill. The returns of funds within a family are subject to correlated idiosyncratic shocks, which are unobservable. Both fund skills and the family skill are unknown latent variables, and are time-varying (except for a special case of constant skills). A fund's alpha increases with its composite skill, and decreases with fund size. Investors estimate funds' composite skills through the observation of the returns of all funds in the family, and allocate wealth across funds, generating flows into and out of funds, as in Berk and Green (2004).

[2]Elton, Gruber, and Green (2007) empirically document a higher correlation of mutual fund returns within than across families

We derive the sensitivities of the optimal estimate of the composite skill to both fund and family performance. These results are in closed-form for fund families that are sufficiently old. For young families, initial conditions matter, and we evaluate numerically the dynamics of these sensitivities over time. Our model highlights two competing effects of good family performance, measured by the average performance of other funds in the family, on the estimate of a member fund's composite skill: a positive common-skill effect and a negative correlated-noise effect. The positive effect arises because family performance contains information about the family skill. The negative effect arises because family performance also contains information about unobservable shocks that affect all funds in the family. While the estimate of a fund's composite skill rises with its own unexpected performance, its response to family performance can be either positive or negative, depending on the relative strength of the common-skill and correlated-noise effects. The sensitivity of a fund's composite skill estimate to family performance increases with the weight of the common component in the composite skill and the number of funds in the family, and decreases with the correlation of noise in fund returns. The sensitivity to fund performance varies with these family characteristics in an opposite way. However, both sensitivities decline as a fund grows older, as investors become more certain about its composite skill.

We explore the implications of optimal cross-fund learning for mutual fund flows, measured as a percentage of fund assets, and find strong empirical support for our model. Combining the CRSP survivor-bias-free mutual fund database with the Morningstar fund manager database, we construct a proxy for the weight of the common component in the composite skill using the overlap rate of managers across funds. We measure the correlation of noise in fund returns by the average pairwise correlation of idiosyncratic returns within families. We find that flows to a member fund on average respond positively to family performance, suggesting the dominance of the common-skill effect. They respond more strongly to family performance when the number of funds is large, and the correlation of fund returns within the family is low. The sensitivity of flows to a fund's own performance declines with fund age, the number of funds in the family, the manager overlap rate across funds, and increases with the correlation of idiosyncratic fund returns. These patterns are consistent with the predictions of our model. Interestingly, for the subsample of funds whose families have a below-average manager overlap rate, offer a below-average number of funds, and show an above-average correlation of idiosyncratic returns, the response of fund flows to family performance is significantly negative. This suggests that the dominance of correlated-noise effect is not only theoretically possible, but also empirically true for a sizable fraction of funds.

Our work contributes to the literature on performance evaluation. Our results extend an important insight of the theory of relative performance evaluation, which forms the foundation of most benchmark-adjusted performance measures. Recognizing that peer performance reveals information about common shocks to multiple agents, the relative performance evaluation literature generally postulates a negative relation between the estimated skill or effort of an agent and the performance of his peers (Holmstrom (1982)). By allowing both unobservable skills and noise to be correlated, our model allows for rich possibilities of cross-unit learning. We show that good peer performance can have either a positive or negative effect on the estimated skill of an agent, and we derive the optimal weight of each signal in the performance evaluation as a function of a number of model parameters. To the best of our knowledge, we are the first to model the common-skill and correlated-noise effects jointly and explicitly. Although our model is developed in the context of mutual fund evaluation, the main insight is relevant for many other settings in which both fundamentals and noise are correlated across units.

Our work also contributes to the understanding of the behavior of mutual fund investors. It is well-known that investors chase good past performance (see, for example, Sirri and Tufano (1998)). In an influential paper, Berk and Green (2004) reconcile such behavior with the well-documented lack of persistence in fund performance. The basic elements of their model are managers with unknown skill, competition among investors, and diseconomies of scale in portfolio management. In a similar environment, Dangl, Wu, and Zechner (2008) model simultaneously mutual fund flows and manager replacement in response to past performance.[3] Both models are silent about cross-fund learning within fund families. We extend the continuous-time structure of Dangl, Wu, and Zechner (2008) to a setup with fund families, and derive equilibrium fund flows as optimal responses of rational investors to both fund and family performance. We present new empirical patterns of mutual fund flows, and show that investors respond to fund and family performance in a manner consistent with optimal learning.

There is a large body of literature on mutual fund performance evaluation.[4] Most methods of evaluation rely solely on a fund's own return or portfolio holding information. Several recent papers propose methods incorporating additional information. For example, Pastor and Stambaugh (2002) estimate the alpha of an actively managed fund using returns on "seemingly unrelated" non-benchmark passive assets. Cohen, Coval, and Pastor (2005) judge a fund manager's skill by the extent to which his or her investment decisions resemble those of managers with distinguished track records. Jones and Shanken (2005) measure performance using the distribution of other funds' alphas in addition to the information in a fund's own return history. Our performance evaluation strategy is in the spirit of this literature, but differs in two important aspects. First, we exploit the information embedded in the performance of a fund's family. Second, while these previous studies focus mainly on the cross-fund learning arising from common skills, we consider both the common-skill and correlated-noise effects, and derive theoretically the determinants of their relative strength.

[3] Other learning-based models for the flow-performance relation include those by Lynch and Musto (2003) and Huang, Wei, and Yan (2007).

[4] See Aragon and Ferson (2006) for an extensive review.

Recently the literature has shown a growing interest in mutual fund families.[5] One paper that is closely related to ours along this line is Nanda, Wang, and Zheng (2004), who find that the stellar performance of one fund has a positive spillover onto the inflows to other funds in the same family. Another is Sialm and Tham (2013), who find that the prior stock price performance of a fund management company predicts money flows of its affiliated funds. Both papers are purely empirical. Our model of cross-fund learning provides a rational explanation for such spillovers, and generates a number of new predictions regarding mutual fund flow sensitivities to fund and family performance, which are supported by the empirical patterns we find in the data.

The paper is organized as follows. Section 1 introduces our model of a mutual fund family. Section 2 derives the rational beliefs of investors about composite skills, conditional on both fund and family performance. Sections 3 derives in closed-form the factors that govern the uncertainty and sensitivities of beliefs to fund and family performance in the long run. Section 4 describes the evolution of these sensitivities over time. Section 5 derives fund flows in equilibrium. Section 6 presents our empirical evidence on fund flows, while Section 7 concludes. The proofs of propositions and corollaries are in the Appendix.

1 A Family of Mutual Funds

We model n actively managed mutual funds within a family. The quality of management is an unobservable factor governing the success or failure of a fund. Quality may vary through time, and is a linear combination of two components, which together form the composite skill θ_t. One part of θ_t is the fund-specific skill. The second part is the common family skill. A fund's alpha is an increasing function of θ_t, and its realized abnormal return is its

[5] See for example, Mamaysky and Spiegel (2002), Massa (2003), Gervais, Lynch, and Musto (2005), Massa, Gaspar, and Matos (2006), Ruenzi and Kempf (2008), Pomorski (2009), Warner and Wu (2011), Khorana and Servaes (2011), Bhattacharya, Lee, and Pool (2013).

alpha plus noise. We calculate a conditional distribution of θ_t for all funds in the family using abnormal fund returns as a continuous signal.

For simplicity, we abstract from managers' market-timing activity and focus only on stock-selection. Funds' incremental abnormal returns, in excess of benchmarks and management fees, $d\mathbf{R}_t$, are given by

$$d\mathbf{R}_t = \boldsymbol{\alpha}_t dt + \boldsymbol{\sigma}_t \mathbf{B} d\mathbf{W}_t. \tag{1}$$

Here, $\boldsymbol{\alpha}_t$ is a $n \times 1$ vector of fund alphas generated by active management, while $\boldsymbol{\sigma}_t \mathbf{B} d\mathbf{W}_t$ is the noise in abnormal returns. The $n \times n$ diagonal matrix $\boldsymbol{\sigma}_t$ represents the scale of funds' idiosyncratic risks. It has elements σ_{it} along the main diagonal, which are the instantaneous volatilities of abnormal returns. Matrix $\mathbf{B}$ is the Cholesky factor of the nonsingular correlation matrix **BB'** that summarizes the instantaneous correlations, ρ_{ij}, of the noise in abnormal returns. $\mathbf{W}_t$ is a vector of standard Brownian motions that are pairwise independent.

A fund's idiosyncratic risk σ_{it} is governed by the scale of the fund's portfolio tilt, which is the difference between the fund's weights in individual securities and the weights of its benchmark portfolio with only systematic risks and zero alpha. A fund with no tilt has $\sigma_{it} = 0$. As a fund increases the scale of a tilt, with the expectation of increasing fund alpha, σ_{it} increases. If two funds i and j follow independent strategies and have orthogonal tilts, the idiosyncratic shocks are uncorrelated, and $\rho_{ij} = 0$. For various reasons noted above, however, we expect funds within a family to follow positively correlated strategies.[6]

As emphasized by Berk and Green (2004), there are diseconomies of scale in active portfolio management. We model fund alphas net of management fees by generalizing the

[6]While we implicitly assume $\rho_{ij} > 0$ when we discuss the correlated-noise effect, our model allows for all $\rho_{ij} \in (-1, 1)$, $i \neq j$, such that $\mathbf{BB}'$ is nonsingular.

specification of Dangl, Wu, and Zechner (2008) to allow for multiple funds:

$$\boldsymbol{\alpha}_t = \boldsymbol{\sigma}_t \boldsymbol{\theta}_t - \gamma \boldsymbol{\sigma}_t \boldsymbol{\sigma}_t \mathbf{A}_t - \mathbf{f}_t, \tag{2}$$

where $\boldsymbol{\theta}_t$, $\mathbf{A}_t$ and $\mathbf{f}_t$ are $n \times 1$ vectors of composite skills, asset sizes and fees, respectively, and $\gamma > 0$ is a parameter characterizing decreasing returns to scale. The i-th element of the first term on the right-hand side of equation (2) is $\sigma_{it}\theta_{it}$, so that alpha increases linearly with the composite skill for a given level of idiosyncratic risk. The i-th element of the second term is $-\gamma\sigma_{it}^2 A_{it}$. Thus, a fund's alpha decreases with its own size, and at a higher rate when a fund is more actively managed, i.e., when σ_{it} is high. Equation (2) captures the idea that the diseconomies of scale rises with the degree of active management. A passively managed fund, such as an index fund, suffers less from the price impact of trades because fund inflows are allocated into a broad set of securities. The equation also implies that the marginal return from taking idiosyncratic risk decreases, especially for large funds. This deters funds from taking unlimited idiosyncratic risk.[7]

Composite skills vector $\boldsymbol{\theta}_t$ is a linear combination of two components:

$$\boldsymbol{\theta}_t \overset{def}{=} (1-\beta)\,\boldsymbol{\theta}_{ft} + \beta\theta_{Ft}\mathbf{1}_n, \tag{3}$$

where $\boldsymbol{\theta}_{ft} \overset{def}{=} \begin{pmatrix} \theta_{f1,t}, & ..., & \theta_{fn,t} \end{pmatrix}'$ is a vector of fund-specific skills, θ_{Ft} is a scaler representing the family skill, and $\beta \in [0,1]$ is the weight of the family skill in the composite skill, $\mathbf{1}_n$ is a n vector of ones. For a fund with no fund-specific skill, $\theta_{it} = 0$. A family with a pool of excellent analysts has large θ_{Ft}. If managers of individual funds work independently of the family resources, $\beta = 0$. We expect individual funds to rely on family resources, i.e.,

[7] Our performance evaluation method only requires $\boldsymbol{\alpha}$ to be linear in the latent variable $\boldsymbol{\theta}$. We specify $\boldsymbol{\alpha}$ explicitly. This allows us to derive equilibrium fund flows in closed-form. See Dangl, Wu, and Zechner (2008) for a more detailed discussion of this specification.

$\beta > 0$. In this case, the fund's alpha increases directly with both $\theta_{fi,t}$ and θ_{Ft}.[8]

Mutual funds operate in a rapidly changing business environment. Past success or experience is no guarantee of future performance. To capture this characteristic of the industry, we assume the unobservable composite skills follow a stochastic process:

$$d\boldsymbol{\theta}_t = k\left(\overline{\boldsymbol{\theta}} - \boldsymbol{\theta}_t\right)dt + \boldsymbol{\Omega} d\mathbf{Z}_t, \tag{4}$$

where the constant k governs the speed at which $\boldsymbol{\theta}_t$ reverts to the long-run mean $\overline{\boldsymbol{\theta}}$, and $\mathbf{Z}_t$ is a vector of $n+1$ pairwise independent standard Brownian motions, each independent of $\mathbf{W}_t$.[9] Volatility coefficients are in the $n \times (n+1)$ matrix

$$\boldsymbol{\Omega} = \left[(1-\beta)\boldsymbol{\omega}_f, \quad \beta\omega_F \mathbf{1}_n\right], \tag{5}$$

where $\boldsymbol{\omega}_f$ is a $n \times n$ diagonal matrix with coefficients $\omega_{fi} \geq 0$ along the main diagonal, representing the instantaneous volatilities of fund skills, while $\omega_F \geq 0$ is the volatility of the family skill. Thus, the stochastic component of an element $d\theta_{it}$ in (4) is $(1-\beta)\omega_{fi}dZ_{i,t} + \beta\omega_F dZ_{n+1,t}$. Denote the instantaneous volatility of the composite skill of fund i by ω_i, and we have

$$\omega_i = \sqrt{(1-\beta)^2\omega_{fi}^2 + \beta^2\omega_F^2}. \tag{6}$$

Equations (4)-(6) nest three important cases. First, when $k = \omega_i = 0$ for all i, composite skills are constant ($\boldsymbol{\theta}_t = \overline{\boldsymbol{\theta}}$). Second, when $k = 0$ and $\omega_i > 0$ for all i, composite skills follow random walks. Finally, when $k > 0$ and $\omega_i > 0$ for all i, composite skills are mean-reverting.

When $\omega_i > 0$ for all i, the instantaneous covariance matrix $\boldsymbol{\Omega}\boldsymbol{\Omega}'$ is positive definite, and

[8] We assume funds rely on family resources to the same degree, so that they share the same β. This can be easily generalized to a case in which the β are heterogeneous across funds.

[9] Kacperczyk, Van Nieuwerburgh, and Veldkamp (2012) provide empirical evidence of time-varying fund manager skill.

the instantaneous correlation of the true composite skills for a pair of funds i and j is

$$\lambda_{ij} \stackrel{def}{=} \frac{\beta^2 \omega_F^2}{\omega_i \omega_j}. \tag{7}$$

This measures the variation in the family skill as a driver of composite skills, relative to the variation in fund skills. It is easy to see that λ_{ij} increases with β, and decreases with the ratios of ω_{fi}/ω_F and ω_{fj}/ω_F. A value $\lambda_{ij} = 0$ indicates either that the family skill is constant, $\omega_F = 0$, or that managers of individual funds work independently, $\beta = 0$. A value $\lambda_{ij} = 1$ indicates instead that the fund skills are constant, $\omega_{fi} = \omega_{fj} = 0$, or that member funds act in concert and rely entirely on the family skill for alpha generation, $\beta = 1$.

2 Evaluation of Composite Skills

Information is symmetric but incomplete. Composite skills $\boldsymbol{\theta}_t$ and the idiosyncratic shocks to returns $d\mathbf{W}_t$ are not observable. Investors form beliefs about the conditional distribution of unobservable composite skills, using the returns of all funds in a family as signals. We now derive the optimal updating of the beliefs.

Substituting for $\boldsymbol{\alpha}_t$ using equation (2), the observable components in equation (1) form a signal $d\boldsymbol{\xi}_t$ of composite skills. This is

$$\begin{aligned} d\boldsymbol{\xi}_t &\stackrel{def}{=} \boldsymbol{\sigma}_t^{-1} \left[d\mathbf{R}_t + \left(\gamma \boldsymbol{\sigma}_t \boldsymbol{\sigma}_t \mathbf{A}_t + \mathbf{f}_t \right) dt \right] \\ &= \boldsymbol{\theta}_t dt + \mathbf{B} d\mathbf{W}_t. \end{aligned} \tag{8}$$

The signal is centered on the drift $\boldsymbol{\theta}_t dt$ and has noise $\mathbf{B}d\mathbf{W}_t$ that is correlated across funds.

At any time t, information is the history of fund returns represented by the filtration $\mathcal{F}_t \stackrel{def}{=} \sigma \left\{ \boldsymbol{\xi}_s \right\}_{s=0}^{t}$. Given a multivariate normal prior distribution with mean vector $\mathbf{m}_0$ and covariance matrix $\mathbf{V}_0$, the conditional distribution of composite skills is also multivariate normal.[10]

Proposition 1. *The conditional mean vector* $\mathbf{m}_t \stackrel{def}{=} E(\boldsymbol{\theta}_\mathbf{t}|\mathcal{F}_t)$ *and the conditional covariance matrix* $\mathbf{V}_t \stackrel{def}{=} Var(\boldsymbol{\theta}_t|\mathcal{F}_t)$ *for* $t \geq 0$ *follow the processes:*

$$d\mathbf{m}_t = k\left(\overline{\boldsymbol{\theta}} - \mathbf{m}_t\right)dt + \mathbf{S}_t d\mathbf{W}_t^{\mathcal{F}}, \tag{9}$$

$$\frac{d\mathbf{V}_t}{dt} = \boldsymbol{\Omega}\boldsymbol{\Omega}' - 2k\mathbf{V}_t - \mathbf{V}_t\left(\mathbf{B}\mathbf{B}'\right)^{-1}\mathbf{V}_t, \tag{10}$$

where

$$\mathbf{S}_t \stackrel{def}{=} \mathbf{V}_t\left(\mathbf{B}\mathbf{B}'\right)^{-1}, \tag{11}$$

$$d\mathbf{W}_t^{\mathcal{F}} \stackrel{def}{=} \left(d\boldsymbol{\xi}_t - \mathbf{m}_t dt\right). \tag{12}$$

When $k = 0$*, equation (10) has the analytic solution:*

$$\mathbf{V}_\mathbf{t} = \mathbf{B}\mathbf{P}_\mathbf{t}\mathbf{B}' + \mathbf{V}^*, \tag{13}$$

where

$$\mathbf{V}^* = \begin{cases} \mathbf{0}_{\mathbf{n}\times\mathbf{n}}, & if \quad \omega_i = 0 \quad for \quad all \quad i, \\ \mathbf{B}\mathbf{D}\boldsymbol{\Pi}^{\mathbf{1/2}}\mathbf{D}'\mathbf{B}', & if \quad \omega_i > 0 \quad for \quad all \quad i, \end{cases} \tag{14}$$

is the constant covariance matrix in the long run (as $t \to \infty$*), and the matrices* $\mathbf{D}, \boldsymbol{\Pi}$*, and* $\mathbf{P}_\mathbf{t}$ *are as defined in Appendix A.1.*

Proof. See Appendix A.1.

The conditional mean $\mathbf{m}_t$ follows a multi-variate Orstein-Uhlenbeck process in equation (9), with long-run mean $\overline{\boldsymbol{\theta}}$. The vector $d\mathbf{W}_t^{\mathcal{F}}$ defined in equation (12) is a vector of correlated

[10] Investor beliefs are conditional distributions for $\boldsymbol{\theta}_t$, which has the same dimension as the observation equation. A conditional distribution can be calculated numerically for individual components θ_i and θ_F. Learning these components separately, which is not pursued in this paper, is important for the hiring and firing decisions in fund families, and investor responses to these decisions, but it is not required to form expectations of fund alphas.

Brownian motions under investors' information set $\mathcal{F}_t$ with correlation matrix $\mathbf{BB}'$, and it is the innovation in the signal process $\boldsymbol{\xi}_t$. Taking the expectation of both sides of equation (2), we have $\mathbf{m}_t = \boldsymbol{\sigma}_t^{-1}\left[E\left(\boldsymbol{\alpha}_t|\mathcal{F}_t\right) + \gamma\boldsymbol{\sigma}_t\boldsymbol{\sigma}_t\mathbf{A}_t + \mathbf{f}_t\right]$. Substituting for $d\boldsymbol{\xi}_t$ using equation (8), equation (12) becomes

$$d\mathbf{W}_t^{\mathcal{F}} = \boldsymbol{\sigma}_t^{-1}\left[d\mathbf{R}_t - E\left(\boldsymbol{\alpha}_t|\mathcal{F}_t\right)dt\right], \tag{15}$$

which shows that $d\mathbf{W}_t^{\mathcal{F}}$ is the vector of unexpected abnormal returns standardized by volatility $\boldsymbol{\sigma}_t$. We refer to $d\mathbf{W}_t^{\mathcal{F}}$ simply as *unexpected performance.*

The matrix $\mathbf{S}_t$ in equation (9) characterizes the response of investors' beliefs to mutual fund performance. An element on the main diagonal of $\mathbf{S}_t$ is the sensitivity of a fund's conditional mean to its own unexpected performance. An off-diagonal element is the sensitivity to the unexpected performance of another fund. Equation (11) show that the sensitivities increase with uncertainty about composite skills, which is described by matrix $\mathbf{V}_t$. Elements on and off the main diagonal of $\mathbf{V}_t$ are conditional variances and covariances of $\boldsymbol{\theta}_t$, respectively. Generally, if composite skills are estimated precisely, $\mathbf{S}_t$ is small and beliefs are insensitive to unexpected performance. If instead little is known about the skills, unexpected performance is an important signal, and investors respond to it strongly.

The analytic solution for $\mathbf{V}_t$ in equation (13) obtains when $k = 0$, i.e., when composite skills are constant or follow a random walk. In the case of mean reversion, i.e., $k > 0$, numerical solutions to equation (10) are easily calculated. In principle, elements of $\mathbf{V}_t$ may either decrease or increase with time, depending on the levels of initial uncertainty $\mathbf{V}_0$ and volatilities of true composite skills ω. However, in reality we expect that $\mathbf{V}_t$ decreases early in the life of a family because investors initially know little about composite skills and their uncertainty declines as they learn over time.

The long-run covariance matrix, $\mathbf{V}^*$, is the solution to equation (10) with $\frac{d\mathbf{V}_t}{dt} = \mathbf{0}_{n\times n}$. In comparison to $\mathbf{V}_t$, $\mathbf{V}^*$ is simple because it is time-independent. It is also independent of prior beliefs. When $k = 0$, it has the simple form given in equation (14). Furthermore, $\mathbf{V}^*$ as the limiting value is a good approximation to $\mathbf{V}_t$ after the passage of enough time, i.e., in old families. For these reasons, we first study the long-run covariance matrix $\mathbf{V}^*$ and sensitivity matrix $\mathbf{S}^* = \mathbf{V}^* (\mathbf{BB}')^{-1}$ in Section 3, and then the time-dependent case in Section 4.

3 Sensitivities of Investor Beliefs in the Long Run

The factors that govern the sensitivities of beliefs to performance are most transparent in the long-run limiting case as $t \to \infty$, in which both the uncertainty about composite skills and the sensitivities of beliefs are constant, meaning that $\mathbf{V}_t = \mathbf{V}^*$ and $\mathbf{S}_t = \mathbf{S}^*$. We study $\mathbf{V}^*$ and $\mathbf{S}^*$ in this section.

For simplicity, we assume here that composite skills follow a random walk (i.e., $k = 0$), and defer the case of mean-reversion to Section 4. This allows us to derive analytic solutions, which are good approximations for the case in which skills revert slowly to their long-run means. We focus on a *homogeneous* n-fund family, in which $\beta_i = \beta \in [0, 1], \omega_{fi} = \omega_f > 0$ for all i (and therefore $\omega_i = \omega$ for all i and $\lambda_{ij} = \lambda \in [0, 1]$ for all $i \neq j$), and $\rho_{ij} = \rho \in \left(\frac{-1}{n-1}, 1\right)$ for all $i \neq j$.[11]

For a homogeneous family, matrix $\mathbf{V}^*$ has identical elements on the main diagonal, say, v_n, and identical elements off the diagonal, say, $\overline{v}_n$.[12] Matrix $\mathbf{S}^*$ has the same structure. As $t \to \infty$, the dynamics of the conditional mean for each fund in equation (9) simplifies to

$$dm_{it} = s_n dW_{it}^{\mathcal{F}} + \overline{s}_n dX_{-i,t}^{\mathcal{F}}, \tag{16}$$

[11] The condition $\frac{-1}{n-1} < \rho < 1$ ensures that the matrix $\mathbf{BB}'$ is nonsingular. For the special case of $n \to \infty$, nonsingularity requires $0 \leq \rho < 1$.

[12] For a scalar variable, we use the subscript n to denote explicitly the number of funds in a family, and the subscript t to indicate time-dependence. We suppress the subscript n if a variable is a vector or matrix.

where s_n and $\overline{s}_n/(n-1)$ are the diagonal and off-diagonal elements of $\mathbf{S}^*$, respectively; $dW_{it}^{\mathcal{F}}$ is the unexpected performance of fund i; and $dX_{-i,t}^{\mathcal{F}} \stackrel{def}{=} \frac{1}{n-1}\sum_{j\neq i} dW_{jt}^{\mathcal{F}}$ is the average unexpected performance of the other funds in the family. Equation (16) has the obvious advantage over equation (9) in that the performance of the other funds is summarized in the single statistic $dX_{-i,t}^{\mathcal{F}}$. We refer to $dX_{-i,t}^{\mathcal{F}}$ as the family performance.

The sensitivity coefficients s_n and $\overline{s}_n$ are functions of the elements v_n and $\overline{v}_n$ of the covariance matrix. Section 3.1 describes v_n and $\overline{v}_n$, while Section 3.2 describes s_n and $\overline{s}_n$.

3.1 Uncertainty About Composite Skills

The long-run uncertainty about composite skills is described by the constant conditional covariance matrix $\mathbf{V}^*$ of the vector of composite skills. Proposition 2 gives the diagonal and off-diagonal elements of $\mathbf{V}^*$, v_n and $\overline{v}_n$, respectively, for a homogeneous n-fund family.

Proposition 2. *Assume that composite skills follow a random walk and that funds in the family are homogeneous. If $\rho \neq \frac{n-2}{n-1} - \lambda$, the long-run conditional variance of each fund's composite skill is*

$$v_n = \omega \sqrt{\frac{K_1^2}{\left(K_\rho - K_\lambda\right)^2 (n-1) + K_1^2}} \ \leq \ \omega, \tag{17}$$

and the long-run conditional covariance of composite skills for each pair of funds is

$$\overline{v}_n = \omega \ \frac{K_2 - K_\rho K_\lambda}{\sqrt{\left(K_\rho - K_\lambda\right)^2 (n-1) + K_1^2}} \frac{|K_1|}{K_1}, \tag{18}$$

where $K_1, K_2, K_\rho, and K_\lambda$ are functions of ρ and λ given in equations (A.16), (A.17) and (A.22), respectively. In particular, if $\rho = \lambda$, then $v_n = \omega$; otherwise, $v_n < \omega$.[13]

Proof. See Appendix A.2

[13] Equations (A.19) and (A.20) give v_n and $\overline{v}_n$ for the case $\rho = \frac{n-2}{n-1} - \lambda$.

To better convey the intuition of these results, we present below two special cases of Proposition 2, one with a two-fund family, another with a family with infinitely many funds.

Corollary 1. *Long-run conditional variance and covariance in two special cases:*

(i) For a two-fund family (n=2), the long-run conditional variance and covariance are, respectively,

$$v_2 = \frac{1}{2}\omega\left(\sqrt{1+\rho}\sqrt{1+\lambda} + \sqrt{1-\rho}\sqrt{1-\lambda}\right), \tag{19}$$

$$\overline{v}_2 = \frac{1}{2}\omega\left(\sqrt{1+\rho}\sqrt{1+\lambda} - \sqrt{1-\rho}\sqrt{1-\lambda}\right). \tag{20}$$

(ii) As $n \to \infty$, the long-run conditional variance and covariance are, respectively,

$$v \stackrel{def}{=} \lim_{n\to\infty} v_n = \omega\left(\sqrt{\rho\lambda} + \sqrt{1-\rho}\sqrt{1-\lambda}\right), \tag{21}$$

$$\overline{v} \stackrel{def}{=} \lim_{n\to\infty} \overline{v}_n = \omega\sqrt{\rho\lambda}. \tag{22}$$

Proof. See Appendix A.3.

Other things equal, the long-run uncertainty about composite skills, v_n, is greatest when the correlation of noise in fund returns, ρ, is equal to the instantaneous correlation of true composite skills, λ. This is most easily seen for the two special cases in Corollary 1, as both conditional variances, v_2 in equation (19) and v in equation (21), are maximized in ρ when $\rho = \lambda$. This result also holds for any family size $n \geq 2$. The intuition is simple. As we demonstrate in Section 3.2, when $\rho = \lambda$, there is no opportunity for cross-fund learning. Investors learn about a fund's composite skill using only the fund's own performance. As a result, the estimate of the composite skill is imprecise. Since family performance does not provide additional information about a member fund in this case, the long-run uncertainty is independent of family size.

The precision of an optimal estimate of composite skills in a fund family, $1/v_n$, increases as ρ deviates from λ. Equation (19) and (21) show, for the two-fund and infinite-fund cases respectively, that precision is maximized when either: (i) the funds rely solely on the family skill ($\lambda = 1$), and the noise in their returns is uncorrelated ($\rho = 0$); or (ii) the funds are independently managed ($\lambda = 0$), and the noise is almost perfectly correlated ($\rho \to 1$).[14] Numerical evaluation of equation (17) shows that this holds for all family sizes $n \geq 2$. The intuition is as follows. In case (i), the composite skill consists of only the family component, and it is the same across funds. Due to zero correlation of noise, a simple average of all funds' returns provides a precise estimate of this common component, especially when the family size is big. As the number of funds in the family goes to infinity, the law of large numbers ensures that the family skill is perfectly revealed. In case (ii), the noise in fund returns is perfectly correlated, while the composite skill of each fund consists only of the fund skill. Because innovations of fund skills are independent, they tend to be averaged out, thus family performance gives a precise estimate of the perfectly-correlated noise. This allows investors to estimate fund skills accurately. As the family size goes to infinity, the average of fund-specific skills goes to the known population mean, thus family performance reveals the noise completely. The difference between a fund's performance and the family performance reveals the fund skill completely.

To better illustrate the properties of the long-run uncertainty v_n, we construct a numerical example. We assume that family skill and fund skill are equally important (β=0.5), and their volatilities are 0.10 and 0.15, respectively. This implies that the instantaneous correlation of composite skills, λ, between two funds is 0.31 (equation (7)). According to Proposition 2, the long-run uncertainty is highest when the correlation of noise in fund returns, ρ, is equal to 0.31, and declines as ρ deviates from 0.31. This pattern shows up very clearly in Figure 1, which plots v_n as a function of ρ for families with 2, 10, 30 and infinitely many funds.

[14] For the family with infinitely many funds, uncertainty goes to zero in these two cases.

The figures also shows that the uncertainty about composite skills is in general lower for bigger families, suggesting that the availability of a larger number of signals allows a more precise estimation of composite skills. However, this is not true when $\rho = \lambda$, in which case the family size is irrelevant for the long-run uncertainty due to the absence of cross-fund learning.

3.2 Sensitivities of Beliefs

After computing the long-run conditional covariance matrix, we can now describe the sensitivities of the optimal estimate of composite skill to a fund's own performance and to family performance in a n-fund family, i.e., the coefficients s_n and $\overline{s}_n$ in equation (16).

Proposition 3. *Assume that composite skills follow a random walk and that funds in the family are homogeneous. In the long run, if $\rho \neq \frac{n-2}{n-1} - \lambda$, the direct sensitivity of the estimate of a fund's composite skill to its own performance is*

$$s_n = v_n \left(1 - \left(1 - \frac{K_\lambda}{K_\rho}\right) \frac{\rho}{\frac{1}{n-1} + \lambda + \rho - 1}\right), \tag{23}$$

and the cross-sensitivity of the estimate to the average performance of other funds is

$$\overline{s}_n = v_n \left(1 - \frac{K_\lambda}{K_\rho}\right) \frac{1}{\frac{1}{n-1} + \lambda + \rho - 1}, \tag{24}$$

where K_ρ and K_λ are functions of ρ and λ given in (A.22). In particular, if $\rho = \lambda$, then $s_n = v_n$ and $\overline{s}_n = 0$.[15]

Proof. See Appendix A.4.

Again, the intuition of these results is easier to understand in two special cases.

Corollary 2. *Long-run sensitivities in two special cases:*

[15] Equation (A.25) and (A.26) give s_n and $\overline{s}_n$ for the case $\rho = \frac{n-2}{n-1} - \lambda$. Note that $K_\rho > 0$ since $\rho \in (\frac{-1}{n-1}, 1)$.

(i) For a two-fund family (n=2), the long-run direct sensitivity and cross-sensitivity are, respectively,

$$s_2 = \frac{1}{2}\omega\left(\frac{\sqrt{1+\lambda}}{\sqrt{1+\rho}} + \frac{\sqrt{1-\lambda}}{\sqrt{1-\rho}}\right), \tag{25}$$

$$\overline{s}_2 = \frac{1}{2}\omega\left(\frac{\sqrt{1+\lambda}}{\sqrt{1+\rho}} - \frac{\sqrt{1-\lambda}}{\sqrt{1-\rho}}\right). \tag{26}$$

(ii) As $n \to \infty$, the long-run direct sensitivity and cross-sensitivity are, respectively,

$$s \stackrel{def}{=} \lim_{n\to\infty} s_n = \omega\frac{\sqrt{1-\lambda}}{\sqrt{1-\rho}}, \tag{27}$$

$$\overline{s} \stackrel{def}{=} \lim_{n\to\infty} \overline{s}_n = \begin{cases} 0, & \textit{if } \rho = \lambda = 0, \\ \omega\left(\frac{\sqrt{\lambda}}{\sqrt{\rho}} - \frac{\sqrt{1-\lambda}}{\sqrt{1-\rho}}\right), & \textit{otherwise.} \end{cases} \tag{28}$$

Proof. See Appendix A.5.

Consider first the two special cases of the Corollary. In each case, the direct sensitivity of the skill estimate to a fund's own performance is nonnegative. s_2 is strictly positive as long as composite skills are stochastic ($\omega > 0$), and s is strictly positive if in addition, $\lambda \neq 1$. This means that unexpectedly good performance by a fund raises investor beliefs about its composite skill. Furthermore, both s_2 and s decrease with λ and increase with ρ, indicating investors put more weight on a fund's own performance when noise in fund returns is highly correlated, and when the instantaneous correlation of composite skills is low (due to either a low β or a low ω_F).

In contrast, the sensitivities of the estimate of the composite skill to family performance, $\overline{s}_2$ and $\overline{s}$, decrease with ρ and increase with λ, and are either positive, negative, or zero, depending on the relative sizes of ρ and λ. This ambiguity comes from two competing effects, one due to the correlation of noise, the other due to the existence of a common component in composite skills. The two-fund family illustrates these two competing forces most transparently.

In the two-fund family with $\rho = 0$, $\bar{s}_2 = \frac{1}{2}\omega\left(\sqrt{1+\lambda} - \sqrt{1-\lambda}\right)$. This measures the pure common-skill effect, and is positive as long as $\lambda > 0$. Similarly, when $\lambda = 0$, we have a measure of the pure correlated-noise effect, $\bar{s}_2 = \frac{1}{2}\omega(\frac{1}{\sqrt{1+\rho}} - \frac{1}{\sqrt{1-\rho}})$, which is positive if $\rho < 0$, and negative if $\rho > 0$. Between these two extreme cases, $\bar{s}_2$ is a mixture of both effects. It is positive when $\lambda > \rho$, suggesting that in the face of high instantaneous correlation of composite skills and low correlation of noise in fund returns, the optimal estimate of the composite skill of one fund puts a positive weight on the performance of the second fund. Alternatively, when $\lambda < \rho$, the correlated-noise effect is dominant and $\bar{s}_2 < 0$. Finally, when $\lambda = \rho$, the two effects offset each other, $\bar{s}_2 = 0$, and the evaluation of a fund's composite skill is entirely based on its own performance.[16] Equation (28) shows that these properties of cross-fund learning are also true as the family size goes to infinity. The common-skill effect dominates the correlated-noise effect if and only if $\lambda > \rho > 0$.[17]

Consider now the general case described by Proposition 3. Equations (23) and (24) clearly demonstrate that both the direct sensitivity s_n and the cross-sensitivity $\bar{s}_n$ are directly linked to the long-run uncertainty v_n. Their magnitudes are linearly increasing with v_n, suggesting investors pay more attention to both fund and family performance when they are more uncertain about skills. For any family size, the ratio of $\bar{s}_n$ to s_n depends only on the instantaneous correlation of true composite skills λ, the correlation of noise in fund returns ρ, and the family size n. The other primitive parameters β, ω, and ω_F, affect this ratio through their impacts on λ (equation (7)).

[16] It is worth noting that a positive cross-sensitivity requires $\lambda > \rho$ only in the long-run case ($t \to \infty$), which is the setting of this section. In the short run, the cross-sensitivity depends on initial conditions and is time-dependent. It may be positive even if λ=0, or if λ is undefined, as in the case of constant skills.

[17] Equation (28) also shows that $\bar{s}$ explodes as $\rho \to 0$, provided $\lambda \neq 0$. This does not imply, however, that investors' beliefs have unbounded volatility. Because noise in fund returns is uncorrelated, and variations of fund skills are independent, both tend to be averaged out as $n \to \infty$. As a result, family skill becomes the only driver of variation in the family performance in this limiting case, and can be learned perfectly. This implies that unexpected family performance under investors' information set, $dX^{\mathcal{F}}_{-i,t}$, goes to zero. It is easy to show that $\bar{s}_n dX^{\mathcal{F}}_{-i,t}$ has finite variance even through $\bar{s}$ explodes.

We plot the sensitivity ratio $\frac{\bar{s}_n}{s_n}$ for alternative family sizes in Figure 2, letting the number of funds in a family, n, vary from 2, 10, 30, to ∞. Panel (a) plots the ratio as a function of β, the weight of family skill in the composite skill, while Panel (b) plots it as a function of ρ, the correlation of noise. For all family sizes, the ratio increases with β, decreases with ρ, and is zero when ρ is equal to the value of λ implied by the values of β, ω_f and ω_F. Furthermore, the impacts of β and ρ on the ratio are progressively stronger as the size of the family grows, indicating that investors put more weight in bigger families on unexpected family performance when they evaluate any single fund. For example, while the ratio of cross-sensitivity to direct sensitivity is never lower than -1, it can be substantially bigger than 1 in large families, indicating that investors react to family performance more strongly than to a fund's own performance. This occurs when either β is high, or ρ is low.

4 Sensitivities of Investor Beliefs Over Time

In this section we study the dynamics of investor sensitivities to fund and family performance, allowing for mean-reversion in true skills. The long-run analysis in the prior section highlights the key determinants of these sensitivities, but it is silent about their evolution over time. In reality, uncertainty is higher early in the life of a fund and declines as investors learn from its track record. As the beliefs about composite skills become more precise, the sensitivities of beliefs to both fund and family performance decline. We investigate the paths of these sensitivities over time for families with different characteristics.

We calculate numerically the covariance matrix $\mathbf{V}_t$ and the sensitivity matrix $\mathbf{S}_t$ given by equations (10) and (11) under reasonable parameter values. For tractability, we assume again, as in Section 3, that funds in a family are homogeneous. As a result, $\mathbf{V}_t$ is fully characterized by a pair of differential equations, one for the diagonal elements v_{nt} and one for the off-diagonal elements $\overline{v}_{nt}$. Also, investors' conditional estimate of composite skills follows equation (16), although now the coefficients, s_{nt} and $\overline{s}_{nt}$, are time-varying.

The parameter values in the base case of our analysis are shown in Table 1. The number of funds in a family is n=10, close to the average number of equity funds per family in the CRSP mutual fund sample summarized in Table 2. The correlation of noise in fund returns is $\rho = 0.2$, approximately equal to our empirical estimate of pair-wise correlation of idiosyncratic fund returns within families (0.18). We set $\beta = 1/2$, which means that fund skill and family skill are equally important in generating alphas. Also, we set the volatilities of fund and family skills, ω_f and ω_F, to 0.15 and 0.10, respectively. These values imply that the instantaneous correlation of composite skills is $\lambda = 0.31$, which is higher than ρ. Therefore from Proposition 3, the long-run cross-sensitivity $\overline{s}_n$ is positive. As we see in Section 6, this is an empirically reasonable scenario for a typical fund. The initial variance of composite skills is $v_{n0} = 0.12$, following Dangl, Wu, and Zechner (2008). This is 33% higher than the maximum long-run variance implied by the values of β, ω_f and ω_F given above in the absence of mean-reversion.[18] The initial covariance of composite skills is $\overline{v}_{n0} = 0.05$, which is near 40% of the initial variance. Finally, the mean-reversion rate of skills is $k = 0.05$.

Figure 3 shows the sensitivities of beliefs to both fund and family performance over time, for 20 years beginning at fund inception. In general, both sensitivities decline over time as investors develop more precise estimates of composite skills. However, since skills are stochastic, investors remain uncertain about composite skills and are sensitive to performance throughout the lives of funds. The figure also shows that, for most cases, the sensitivities approach their long-run levels after about 10 years. Therefore, our results about the long-run sensitivities in Section 3 are very useful.

Panels (a) and (b) illustrate the influence of the weight of family skill in the composite

[18] The maximum long-run variance is obtained when $\rho = \lambda = 0.31$ according to Proposition 2, and is equal to 0.09 ($= \sqrt{0.5^2 * 0.15^2 + 0.5^2 * 0.10^2}$).

skill by fixing other parameter values at their base levels. Since this parameter does not enter the initial sensitivity matrix $\mathbf{S_0}$ directly, it has no impact on the sensitivities at time zero, but its effects become increasingly more pronounced over time.[19] Consistent with the long-run results in Proposition 3, as funds rely more on the family skill in alpha generation (β varies from 0.3 to 0.7), beliefs become less sensitive to fund performance and more responsive to family performance. When the reliance on family skill is low (β=0.3), the sensitivity to family performance becomes negative as a fund grows older, indicating the dominance effect of noise correlation.

Panels (c) and (d) illustrate the effects of changes in the correlation of noise in fund returns, ρ, for a family of 10 funds. Like family size, the correlation ρ tends to have opposite effects on the direct sensitivity and the cross-sensitivity. As it increases from 0.05 to 0.35, the sensitivity of beliefs to fund performance increases, except for a short period of time early in the fund's life, while the sensitivity to family performance decreases sharply, from about 0.3 to almost zero for a new fund. This pattern is consistent with Proposition 3. It suggests that the cross-fund learning is significantly affected by the correlation of noise in fund performance. In fact, when ρ is high, the positive common-skill effect of family performance on the estimate of composite skill is largely offset, or even reversed, by the negative correlated-noise effect, and s_{nt} becomes negative for as funds grow older.

Panels (e) and (f) illustrate the effects of family size. As the number of funds in the family increases from 2 to 30, the sensitivity of beliefs to fund performance decreases, while the sensitivity to family performance increases. Equation (31) then implies that as family size increases, the sensitivity of flows to fund performance declines, while the sensitivity to family performance increases. This result is easy to understand. Family performance contains less

[19] We take the initial covariance $\mathbf{V_0}$ as given. In a more general setup, one would expect the ratio $\bar{v}_{n0}/v_{n0}$ to increase with β: as the weight of the common component increases, the correlation between composite skills under the initial information set should also increase. Therefore, one would expect that the sensitivity to family performance increases with β at time 0 as well.

noise and more information about family skill when it represents a large number of funds, so it generates a stronger response in large families. Panel (b) also suggests that the sensitivity to family performance declines faster over time in bigger families. This is because investors learn faster about the family skill in the presence of a larger number of signals. In the long run, even though the uncertainty about the composite skill is lower for larger families (as shown in Figure 1), the sensitivity to family performance is stronger. This suggests that the impact of lower uncertainty is more than offset by the higher information quality of the performance of large families.

While the parameter values we use to draw Figure 3 are mainly illustrative, the general patterns are rather robust. We experiment with a wide range of alternative parameterizations and find similar results. The direct sensitivity s_{nt} declines over time as long as the initial uncertainty v_{n0} is above the long-run variance v_n. The cross-sensitivity $\bar{s}_{nt}$ declines over time as long as $\bar{v}_{n0}/v_{n0}$ is not too small. Higher mean-reversion rate k reduces the long-run uncertainty, and therefore reduces both s_{nt} and $\bar{s}_{nt}$ for all $t > 0$, but it has little effect on the variation of these sensitivities across β, ρ and n. For the special case with constant skills ($\omega_f = \omega_F = k = 0$), Panels (c) through (f) in Figure 3 are largely unchanged except that the long-run sensitivities converge to zero. Since the conditional covariance matrix $\mathbf{V}_t$ is independent of β in this case, as one can see from equation (10) by setting $\mathbf{\Omega} = 0$, the sensitivities s_{nt} and $\bar{s}_{nt}$ are also independent of β. However, if one takes into account the fact that β and the ratio $\bar{v}_{n0}/v_{n0}$ should in general be positively correlated, a large β should still lower the direct sensitivity s_{nt} and increases the cross-sensitivity $\bar{s}_{nt}$.

5 Fund Flows in Equilibrium

We now investigate the implications of optimal cross-fund learning for the dynamics of fund flows, in an environment mimicking that of Berk and Green (2004). Investors provide capital to mutual funds without transaction costs. They direct assets toward funds with positive expected alpha, net of fees, and pull assets from funds with negative expected net alpha. In equilibrium, the size of fund i is constantly adjusted to satisfy $E(\alpha_{it}|\mathcal{F}_t)$ =0, meaning that the conditional expected alpha net of fees is zero. Equation (2) then gives

$$A_{it} = \frac{1}{\gamma}\left(\frac{m_{it}}{\sigma_{it}} - \frac{f_{it}}{\sigma_{it}^2}\right). \tag{29}$$

A mutual fund family maximizes total fee income $\mathbf{f}'\mathbf{A}$, setting optimal fee ratios and idiosyncratic volatilities. The optimal quantities for fund i satisfy

$$\frac{f_{it}}{\sigma_{it}} = \frac{1}{2}m_{it}. \tag{30}$$

The ratio on the left-hand side is determined in equilibrium, but neither the fee nor the idiosyncratic volatility is unique. A fund may set a high fee, attract a low level of assets, and take large positions in mispriced assets. Or, it may set a low fee, attract a large amount of inflows, and stick closely to a benchmark portfolio. Provided that the fund's fee and idiosyncratic risk satisfy equation (30), the total fee income is the same in either case. We follow Dangl, Wu, and Zechner (2008) and assume, without loss of generality, that the family sets constant fees, $\mathbf{f} = (f_i)$. Because $\sigma_{it} > 0$, equation (29) implies that a fund is viable, i.e., it has $A_{it} > 0$ and earns a positive fee, only if the estimated composite skill m_t is positive. Otherwise, the fund is either reorganized or closed.

Equations (29) and (30) determine the equilibrium size of a fund. For $m_{it} > 0$, fund size is a convex function of the estimate of the composite skill:

$$A_{it} = \frac{m_{it}^2}{4\gamma f_i},$$

and, using Ito's lemma and equation (9), the instantaneous growth rate of assets is

$$\begin{aligned} \frac{dA_{it}}{A_{it}} &= 2\frac{dm_{it}}{m_{it}} + \frac{(dm_{it})^2}{m_{it}^2} \\ &= 2k\frac{1}{m_{it}}\left(\overline{\theta}_i - m_{it}\right)dt + \frac{1}{m_{it}^2}\mathbf{S}_{it}\mathbf{B}\mathbf{B}'\mathbf{S}_{it}'dt + 2\frac{1}{m_{it}}\mathbf{S}_{it}d\mathbf{W}_t^{\mathcal{F}}. \end{aligned} \tag{31}$$

where $\mathbf{S}_{it}$ is the i-th row of $\mathbf{S}_t$.[20] By writing $\mathbf{S}_{it}d\mathbf{W}_t^{\mathcal{F}} = \sum_j s_{ijt}dW_{jt}^{\mathcal{F}}$, we see that one fund's asset growth rate responds to the performance of all funds in the family. If $s_{ijt} > 0$, unexpectedly good performance by fund j increases the size of fund i, while if $s_{ijt} < 0$, the relation is negative.

Investor beliefs and fund flows respond continuously to unexpected performance $d\mathbf{W}_t^{\mathcal{F}}$. By substituting the equilibrium condition $E\left(\boldsymbol{\alpha}_\mathbf{t}|\mathcal{F}_t\right)=0$ into equation (15), we see that

$$d\mathbf{W}_t^{\mathcal{F}} = \boldsymbol{\sigma}_t^{-1}d\mathbf{R}. \tag{32}$$

That is, in equilibrium, the unexpected performance is the vector of abnormal returns normalized by idiosyncratic volatilities. Any nonzero abnormal return is a surprise under the investors' information set, and leads to a revision of beliefs and a response of fund flows.

6 Empirical Analysis

Since fund size is determined by investor beliefs about the composite skill in the Berk-Green equilibrium, our results in Sections 3 and 4 on the sensitivities of investor beliefs translate directly into predictions about the response of mutual fund flows to unexpected performance, $d\mathbf{W}_t^{\mathcal{F}}$, given in equation (32). According to equation (31), the magnitude of the flow response, measured as a percentage of assets, is proportional to the sensitivity vector

[20]The first term in the second line is a drift of fund size due to the mean-reversion of skills. The second term is a positive drift due to the convex relation between the fund size and the conditional mean of composite skills.

$\mathbf{S}_{it}$. Furthermore, it is inversely related to the conditional estimate of the composite skill, m_{it}, which is positively related to the equilibrium fund size. Our model thus yields the following hypotheses about fund flows:

H1: A higher weight of the common component in the composite skill reduces the sensitivity of flows to fund performance, and increases the sensitivity to family performance;

H2: A higher correlation of noise in fund returns within a family increases the sensitivity of flows to fund performance, and reduces the sensitivity to family performance;

H3: A larger number of funds in the family reduces the sensitivity of flows to fund performance, and increases the sensitivity to family performance;

H4: Fund flows become less sensitive to both fund performance and family performance as funds grow larger and grow older.

We test these predictions of our model in this section.

6.1 Data and Summary Statistics

Our data come primarily from the CRSP survivor-bias-free mutual fund database. Our sample covers the period from January 1999 through December 2011. We focus on domestic equity funds. The advantage of using this relatively homogeneous sample is that it allows us to use the same asset pricing model to estimate the abnormal returns of all funds.[21] The data are at the share class level. We use the MFLINKS database to aggregate the data to the portfolio (i.e., fund) level. A fund's total net asset value (TNA) is the sum across

[21] We select all funds in the following Lipper classes: Large-Cap Core, Large-Cap Growth, Large-Cap Value, Mid-Cap Core, Mid-Cap Growth, Mid-Cap Value, Small-Cap Core, Small-Cap Growth, Small-Cap Value, Multi-Cap Core, Multi-Cap Growth, Multi-Cap Value. The Lipper fund classification information begins in year 1999. The database uses different classification systems for years prior to 1999. Furthermore, for most funds, the management company code, a data item we use to identify the fund family, begins in 1999.

share classes. Its return and expense ratio are the averages, weighted by the prior-period asset share of each class. Fund age is defined as the number of years since the inception of the oldest share class. A fund's family affiliation is identified by a management company code, together with a company name. We exclude the time period before a fund's total net asset value reaches five million dollars, and exclude funds with fewer than 36 monthly return observations. The final sample consists of 2459 funds, affiliated with 686 fund families.[22]

Table 2 reports summary statistics of our sample at the fund-month level. The family-level characteristics are assigned to all member funds. For example, the last row of the table reports the average number of funds in a family, which is 12.2. This is the average fund family size across all fund-month observations.

Fund flow is defined as the difference between the monthly growth rate of the fund's assets (TNA) and the monthly return. To mitigate the effects of extreme observations and potential data errors, we exclude observations below the 1st or above the 99th percentiles of the full sample. Our results are qualitatively the same without this step. Funds on average receive an inflow of 0.28% per month during the sample period.

Investors in our model learn by observing funds' abnormal returns normalized by idiosyncratic volatilities (see equations (9) and (32)). These signals are known as the Treynor and Black (1973) information ratio, which is widely used in portfolio performance evaluation. To stay closely to our model, we use information ratio as our empirical measure of fund performance. A nice feature of this measure is that it explicitly accounts for the noisiness of a fund's abnormal returns. We construct monthly series of information ratios using two standard models: the Fama-French three-factor model and the Carhart (1997) four-factor model. A fund's performance, *Perf*, is the ratio of it's factor-model alpha and the standard deviation of it's model residuals, each estimated using the 36 monthly observations up to

[22] 789 funds in our sample changed family affiliation during the sample period. For example, nine Merrill Lynch funds switch to their family affiliation to Blackrock in the first quarter of 2007.

the current month.[23]

Our mean estimates of *Perf* using the three- and four-factor models are very similar. The average fund earns a negative alpha of approximately ten basis points per month, net of expenses, and it exhibits an idiosyncratic volatility of 1.4% per month, or about 5% per annum. The average monthly information ratio is -0.10, which corresponds to an annualized ratio of -0.35.[24]

In each month of the sample, family performance, *FamPerf*, is calculated for each fund as the average of *Perf* across all funds in the family, excluding the fund itself. We also compute the average pairwise correlation of idiosyncratic returns, *Rho*, between the member funds of a family, and we assign this average value to all member funds. The correlations are also estimated with rolling windows of 36 months. Only the funds affiliated with a single family during the 36-month estimation period enter the calculations of *FamPerf* and *Rho*.

Figure 4 plots the monthly average pairwise correlations of idiosyncratic returns across all pairs of funds within a family. For comparison, we also plot the average pairwise correlations of idiosyncratic returns between 278 standalone funds in our sample. These funds do not belong to a family that offers multiple domestic equity funds. The results estimated using the three- and four-factor models are very similar. The average correlation between stand-alone funds across all months is only 3.4%, while the average correlation between funds within a family is 3.5 times higher, at a level of 15.2%.[25] The difference of 11.8% between the

[23]Factor returns and the Treasury rates used for calculating excess returns are obtained from the website of Professor Ken French at Dartmouth College. *Perf* is calculated only for funds that have been affiliated with a given fund family for at least 36 months and have at least 30 return observations in the 36-month rolling period.

[24]The fact that mutual funds on average earn a negative alpha net of expenses poses a challenge to the Berk and Green (2004) equilibrium, as argued by Fama and French (2010). One of the reasons for such underperformance is that the benchmark returns do not account for portfolio transaction costs.

[25]These numbers are the averages of the three- and four-factor models. The 15.2% within-family correlation is slightly different from the corresponding numbers in Table 2 because of different averaging procedures. Here we first calculate all the pairwise correlations in a given month (separately for pairs within the same family, and pairs of standalone funds), and then average across months. In Table 2, we first calculate the average pairwise correlation for each family in each month, and then average the family-wide correlations across all fund-months in the sample.

within-family and across-family correlations is stable and persistent. This is evidence of a strong family effect in fund returns. It highlights the importance of the common noise in idiosyncratic returns of funds within families.

Another key parameter in our model is the weight of a family's common skill in a fund's composite skill, β. Unfortunately, this weight cannot be measured directly. We therefore construct a proxy for it using the average manager overlap rate across funds. The idea is straightforward. If each fund is managed by a unique manager or management team, then the common component in member funds' composite skills is relatively low. By contrast, if all funds in the same family are managed by the same manager or management team, then the common component is high. To quantify such differences, for each pair of funds within the same family, we define a pairwise manager overlap rate as the the number of managers managing both funds divided by the average number of managers across the two funds. We then average this ratio across all pairs of member funds to obtain the manager overlap rate for a family. We acknowledge that this ratio may not measure the absolute level of β accurately, as there are other types of family and fund-specific resources used in the alpha-generating process. Nevertheless, it captures an important source of variation in β across families.

While the CRSP mutual fund database contains a data item recording the names of fund managers, it has important limitations. With only a few exceptions, all funds with more than three managers are recorded as "Team-Managed", for which manager names are not reported. Massa, Reuter, and Zitzewitz (2010) compare the CRSP and Morningstar mutual fund manager data, and find that Morningstar does a significantly better job in capturing what funds disclose in their SEC filings. A random check of a small number of fund prospectuses confirms their results. We therefore use Morningstar as our source of manager data, as they do in their study. The database contains a unique code for each manager, which greatly facilitates the identification of managers. It records the start and end dates of each manager at each fund. Using CUSIP codes, fund tickers and fund names, we are able to match 95% of the fund-month observations in our CRSP sample to Morningstar.[26]

Table 2 shows that the average manager overlap rate across fund months is 0.19, with a standard deviation of 0.28. Not surprisingly, we find that the manager overlap rate tends to be lower in families with a large number of funds. This is not a problem for multivariate regressions in which family size effects are explicitly accounted for. However, if the manager overlap rate is used as a single sorting criterion, an adjustment for family size is important.

6.2 Full-Sample Results

We now test the predictions of our model. We use the Fama-MacBeth procedure to investigate the cross-sectional patterns in investor responses to fund and family performance. Each month, fund flows are regressed on *Perf* and *FamPerf*, fund and family characteristics, and the interaction terms between these two sets of variables. The fund characteristics include: expense ratio, *Expense*; fund size, $Log(TNA)$; and fund age, $Log(AGE)$. The family characteristics include: family size, $Log(N)$; manager overlap rate, denoted by *Beta*; and the correlation of idiosyncratic returns in a family, *Rho*. Hypotheses **H1-H4** are statements about the signs of the coefficients on the interaction terms. We also include the square of fund performance, $Perf^2$, as a regressor to account for the convexity of the flow-performance relation that shows up in the drift of equation (31). Fund age and fund size are highly correlated, so we investigate their effects both separately and jointly. To account for potential changes of investor preferences for different types of funds over time, fund flows are adjusted by subtracting the contemporaneous mean of all funds in the same Lipper class. Further-

[26]We treat manager data as missing if manager names are unavailable from Morninstar. For the years 1999 to 2004, about 5% of the monthly observations of the manager name variable are recorded as "Team-managed" in Morningstar. This ratio is lower than what is reported in Massa, Reuter, and Zitzewitz (2010), as their sample also includes bond funds and international funds. From 2005 to 2011, this ratio decreases to 0.2%. This is because the SEC introduced a new rule in 2004, which requires funds to disclose the identities of all team members. In contrast, CRSP records 18% of funds as "team-managed" for the first period, and 32% for the second.

more, to aid the interpretation of the regression results, each fund and family characteristic is adjusted by subtracting the contemporaneous sample mean. Therefore, for a fund whose characteristics are equal to their contemporaneous means, the sensitivities of flows to fund and family performance are simply given by the coefficients of $Perf$, $Perf^2$ and $FamPerf$.

There are altogether 120 monthly cross-sectional regressions for each model specification. Table 3 reports the time series averages of the coefficients, and the t-statistics with Newey-West correction for autocorrelation (with three lags). The first three columns report the results using the three-factor model to estimate *Perf*, *FamPerf*, *Rho*, while last three columns correspond to the four-factor model. The two sets of results are very similar, suggesting that the results are robust to the asset pricing model chosen for performance measurement.

The estimated coefficients on *Perf* and $Perf^2$ are positive and highly significant. The positive coefficient on $Perf^2$ confirms the well-documented convex relation between flows and fund performance. The coefficient on family performance, *FamPerf*, is also positive, and significant at the 1% level. It follows that, for a fund whose fund and family characteristics are the mean levels, investors respond positively to family performance, i.e., the common-skill effect dominates the correlated-noise effect. This is consistent with the positive spillover found in Nanda, Wang, and Zheng (2004).

Hypothesis **H1** receives mixed support from the data. Fund flows are less sensitive to fund performance when the manager overlap rate (*Beta*) is high. The coefficient on *Beta**Perf* is negative and statistically significant at the 1% level in each column, suggesting investors rely less on an individual fund's performance to learn about its composite skill, as our model predicts. On the other hand, our prediction that the manager overlap rate increases the sensitivity of fund flows to family performance does not receive support from the data, as the coefficient on *Beta**FamPerf* is insignificant.

The predictions regarding the impacts of the idiosyncratic return correlation (**H2**) receive strong support from the data. Fund flows are more sensitive to fund performance and less sensitive to family performance for families with highly correlated idiosyncratic returns. The coefficient on *Rho*Perf* is positive and significant at the 1% level in each column, while the coefficient on *Rho*FamPerf* is significantly negative in each column. The coefficient of the interaction term suggests a strong negative impact of *Rho* on the sensitivity of fund flows to family performance. Take model (6) as an example. The coefficients on *FamPerf* and *Rho*FamPerf* are 0.754 and -1.290, respectively. These coefficients imply that other things equal, as *Rho* increases from the sample mean by one standard deviation (0.177, as reported in Table 2) , the sensitivity of fund flows to family performance decreases from 0.754 to 0.526 ($= 0.754 - 0.177 * 1.290$), a significant drop of 30%. This suggests that the common-skill effect is significantly offset by the correlated-noise effect as the correlation of idiosyncratic fund returns increases.

The predictions regarding the impacts family size (**H3**) are also strongly supported by the data. Fund flows are less sensitive to fund performance and more sensitive to family performance in families with many funds. The coefficients on *Log*(N)**Perf* and *Log*(N)**FamPerf* are negative and positive, respectively, in each column of the table, and all are statistically significant. These results support our model. The performance of a large family is a relatively precise signal about the family skill, so it induces a strong response of fund flows. At the same time, the response to fund performance is weaker.

Hypothesis **H4** is also supported by the data. Fund flows are less sensitive to fund performance as a fund grows older and larger. The coefficients on the interactions *Log*(TNA)**Perf* and *Log*(Age)**Perf* are negative and highly significant when fund size and age enter the regression separately. When they enter jointly, as in model (3) and (6), the negative age effect drives out the significance of the size effect. This highlights the decline of uncertainty about composite skills as fund grows older. The sensitivity of fund flows sensitivity to family performance also declines with fund age and fund size, as Hypothesis **H4** predicts. While the age effect is only marginally significant in one out of four model, the negative impact of fund size on the flow sensitivity to family performance is significant in all models.

The coefficients on $Log(TNA)$ and $Log(Age)$ are negative when these variables enter the regression separately, indicating that larger and older funds on average attract less inflows as a percentage of their asset sizes, independent of their performance. This is consistent with our model, in which the positive drift of fund size comes from the convex relation between the estimated composite skill and the equilibrium size. As the uncertainty declines over time, the drift becomes weaker.

We conduct a number of robustness checks. For example, we measure fund flows by asset growth rate without adjusting for fund returns, and we use the multi-factor alpha instead of the information ratio as the performance measure. We also use several different proxies for the weight of the common component in the composite skill. For example, we use the fraction of fund pairs managed by an identical manager or management team. We also use the ratio of the number of management teams to the number of funds in the family as an inverse proxy for β. Our results are similar in these alternative tests.

Overall, the full-sample results demonstrate that investors use both fund and family performance when they allocate money across mutual funds, and that they do so in a manner that is largely consistent with our model of rational learning. Each of the hypothesis **H1-H4** receives empirical support. The response of fund flows to fund performance is strongest when funds are young and small, when the number of funds in the family is small, when the manager overlap rate is low, and when the correlation of idiosyncratic returns is high. The response to family performance is strongest when funds are small, when their family is large, and when the correlation of idiosyncratic returns is low. One notable failure is that we find no evidence that a high manager overlap rate within a family increases the sensitivity of flows to family performance. Our results thus demonstrate both the usefulness and the limitation of a model of rational learning to explain mutual fund flows.

6.3 Subsample Analysis

Our model demonstrates that the cross-sensitivity of fund flows to family performance can be either positive or negative, depending on family characteristics. Table 3 shows that this cross-sensitivity is positive for a fund whose characteristics are at the mean levels. This raises an interesting question: While a negative cross-sensitivity is theoretically possible, is it empirically relevant? Extrapolations using the regression results in Table 3 suggest that a negative cross-sensitivity can be obtained in families with certain characteristics. However, such extrapolations do not tell us how prevalent such families are in the data.

To address this question, we apply the Fama-MacBeth procedure separately to two subsamples with opposite families characteristics. Sample 1 represents funds in families that are likely to have a strong correlated-noise effect and a weak common-skill effect. According to our model, these families have a low common skill component β, a high correlation of noise ρ, and a small number of firms n. Therefore, we construct this sample by selecting all funds with the following family characteristics: (1) a manager overlap rate that, after adjusting for family size, is below the contemporaneous mean across all funds;[27] (2) an idiosyncratic return correlation that is above the contemporaneous mean across all funds; (3) a family size (measured by the number of funds) that is below the contemporaneous mean across all funds. Approximately 13% of fund-month observations satisfying these criteria. If investors behave as our model predicts, we should find that fund flows respond negatively to family performance in this sample. Furthermore, fund flows should respond more strongly to fund performance in this sample than in the full sample.

In contrast, sample 2 represents funds that are likely to show a strong common-skill effect and a weak correlated-noise effect. These are funds in families with above-average adjusted manager overlap rate and family size, and below-average idiosyncratic return correlation.

[27]Since manager overlap rate is negatively related to the number of funds in the family, we first run a regression of this rate on $log(N)$ month by month at the family level, and use the residuals from this regression to classify funds.

Our model predicts that fund flows in sample 2 respond more strongly to family performance, and less strongly to fund performance, in comparison to those in the full sample.

The results in Table 4 strongly confirm these predictions. Panels (A) and (B) report the results from the three- and the four-factor models, respectively. The results are very similar across panels, but each panel shows striking differences between the two subsamples. Most interestingly, for sample 1, the fund flow sensitivity to family performance is negative in each model of the two panels, significant at the 1% level in five out of six cases. Consider Panel (A) as an example. The parsimonious specification of model (1) shows that unconditionally, when the average information ratio of all other funds in a family increases by one unit, flows to a member fund decease by 0.862%. Model (2) shows that for funds whose age is at the cross-sectional mean level (i.e., log(Age)=0), this sensitivity is -0.638%. Model (3) shows a cross-sensitivity of -1.055% for a fund whose size is at the mean level. These results demonstrate the dominance of common-noise effect in sample 1, which confirms the empirical relevance of a novel prediction of our model.

The sensitivity of flows to family performance in sample 2 is strongly positive, as our model predicts. Its magnitude is two to three times as big as that observed in the full sample. Furthermore, the flow sensitivities to fund performance in these two subsample also behave as predicted. Compared to the full sample, the coefficient of $Perf$ is about 20% higher in sample 1 (comparing, for example, models (2) and (3) in Panel A with models (1) and (2) in Table 3, respectively), and about 12% to 20% lower in sample 2.

The contrasting results from the two subsamples not only provide strong further support to the cross-sectional predictions of our model, but also demonstrate the economic importance of both the common-skill and common-noise effects. Either one can be very strong when the other counteracting effect is weak. In particular, the results from sample 1 show that the dominance of the correlated-noise effect is not only theoretically possible, but also empirically true for a sizable fraction of funds.

7 Conclusion

The performance of a mutual fund depends on both the fund-specific characteristics and the quality of the common resources of its family. Using family resources in the alpha-generating process introduces a common component in member funds' unobservable skills. It also induces a positive correlation of noise in idiosyncratic fund returns within a family. These observations suggest rich possibilities of cross-fund learning within a family. Building on this idea, we develop a model that characterizes the optimal evaluation of a fund's composite skill based on its own performance and the family performance.

Our model highlights two potential impacts of one fund's performance on the optimal estimate of the composite skill of another fund in the family. When one fund performs well, it indicates the quality of the common resource is high. This is good news about the composite skill of another fund. We call this positive effect the "common-skill effect." When a fund is doing well, this also suggests that it may have had some good luck. Due to the correlation of the noise in returns, we may also attribute a greater portion of another fund's performance to good luck. This is bad news about the composite skill of another fund. We call this negative effect the "correlated-noise effect." The overall effect of one fund's performance on the skill estimate of another fund depends then on the relative strength of these two opposite effects. Our theoretical analysis pins down the key variables that determine the sensitivities of beliefs about composite skills to both fund and family performance, including the number of funds in the family, the importance of common skill in alpha generation, the correlation of noise in fund returns, as well as fund size and fund age.

We empirically test the implication of optimal cross-fund learning for mutual fund flows, and find strong support for our model. Good family performance on average has a positive effect on fund flows to a member fund, suggesting the dominance of the common-skill effect. Its has a stronger impact in larger families, and families with a lower correlation of idiosyncratic returns. Interestingly, for the subsample of funds whose families have a below-average manager overlap rate, offer a below-average number of funds, and show an above-average correlation of idiosyncratic returns, the response of fund flows to family performance is significantly negative. This suggests that the correlated-noise effect dominates the common-skill effect in a sizable fraction of our sample. The sensitivity of flows to a fund's own performance decreases with fund age, family size, and the manager overlap rate across funds, but increases with the correlation of idiosyncratic returns within families, as our model predicts.

We have focused on the Berk and Green (2004) equilibrium of the mutual fund industry with no frictions in fund flows. As a result, there is no predictability in mutual fund returns. Family performance relevant for the optimal updating of beliefs is immediately reflected in fund flows. In practice, it is likely that transaction costs in reallocating money across funds cause temporary deviations from such an equilibrium. Family performance is then useful in predicting both fund returns and fund flows. This is a fruitful avenue for future research.

While our model is about the evaluation of mutual funds in a family setup, its basic insight is relevant in many other situations in which the performance of multiple units depends on both unobservable common fundamentals and noise correlated across the units. For example, it may help to explain some puzzling empirical findings about CEO compensation and turnovers. Empirical studies show that many components of executive compensation, such as stock or stock options, depend on absolute performance rather than performance relative to peers (Murphy (1999)). Recent studies also find that forced CEO turnovers are negatively related to both peer-adjusted CEO performance and peer performance (Jenter and Kanaan (2010)). These findings contradict the simple model of relative performance evaluation. Accounting for both the common-skill and correlated-noise effects may lead to a better understanding of these results. Other applications of our modeling framework include the evaluation of mutual funds or hedge funds with the same investment style, stocks with similar characteristics, or markets in the geographic region.

A Appendix

A.1 Proof of Proposition 1

Equations (9)-(12) follow from Theorem 12.7 of Lipster and Shiryaev (2001), using (4) and (8) as the state and observation equations, respectively.

For the analytic solution to the Ricatti equation (10) given in equation (13) for $k = 0$, we consider two cases: (i) the unknown composite skills are constant, so that $\mathbf{\Omega\Omega}' = \mathbf{0}_{n\times n}$, and (ii) the unknown composite skills follow a random walk, so that $\mathbf{\Omega\Omega}'$ is a positive definite matrix. In either case, in the long run, the covariance matrix of composite skills $\mathbf{V}^*$ is defined by the equation:

$$\mathbf{0}_{n\times n} = \mathbf{\Omega\Omega}' - \mathbf{V}^* \left(\mathbf{BB}'\right)^{-1} \mathbf{V}^*. \tag{A.1}$$

The solution to this equation can be written as

$$\mathbf{V}^* = \begin{cases} \mathbf{0_{n\times n}}, & case \quad (i), \\ \mathbf{BD\Pi}^{1/2}\mathbf{D}'\mathbf{B}', & case \quad (ii), \end{cases} \tag{A.2}$$

where $\mathbf{D}$ and $\mathbf{\Pi}$ are the $n \times n$ matrix of orthonormal eigenvectors and the diagonal matrix of eigenvalues, respectively, of the symmetric, positive-definite matrix

$$\mathbf{D\Pi D}' = \mathbf{B^{-1}\Omega\Omega}'\mathbf{B}'^{-1}. \tag{A.3}$$

A general solution to equation (10) is written

$$\mathbf{V}_t = \mathbf{BP}_t\mathbf{B}' + \mathbf{V}^*, \tag{A.4}$$

where $\mathbf{P}_t$ is to be found. Substitution of (A.4) into equation (10) gives a homogeneous equation:

$$\frac{d\mathbf{P}_t}{dt} = \begin{cases} -\mathbf{P}_t\mathbf{P}_t, & case \quad (i), \\ -\mathbf{P}_t\mathbf{D}\mathbf{\Pi}^{1/2}\mathbf{D}' - \mathbf{D}\mathbf{\Pi}^{1/2}\mathbf{D}'\mathbf{P}_t - \mathbf{P}_t\mathbf{P}_t, & case \quad (ii). \end{cases} \tag{A.5}$$

For case (i), it is easy to show that[28]

$$\mathbf{P}_t = \mathbf{Q}_t^{-1}, \tag{A.6}$$

where the matrix $\mathbf{Q_t}$ satisfies $\mathbf{dQ}_t = \mathbf{I}_{n\times n}$, and has the elements

$$Q_{ij}(t) = \begin{cases} Q_{ij}(0) + t, & i = j, \\ Q_{ij}(0), & i \neq j. \end{cases} \tag{A.7}$$

For case (ii), we define the matrix $\mathbf{Q}_t$ by

$$\mathbf{P}_t = \mathbf{D}\mathbf{Q}_t^{-1}\mathbf{D}'. \tag{A.8}$$

Thus, equation (A.5) implies that $\mathbf{Q}_t$ solves the linear equation

$$\frac{d\mathbf{Q}_t}{dt} = \mathbf{\Pi}^{1/2}\mathbf{Q}_t + \mathbf{Q}_t\mathbf{\Pi}^{1/2} + \mathbf{I}_{n\times n}. \tag{A.9}$$

A solution to this equation has the individual elements

$$Q_{ij}(t) = \begin{cases} e^{2\sqrt{\pi_i}t}Q_{ij}(0) + \frac{1}{2\sqrt{\pi_i}}\left(e^{2\sqrt{\pi_i}t} - 1\right), & i = j, \\ e^{\left(\sqrt{\pi_i}+\sqrt{\pi_j}\right)t}Q_{ij}(0), & i \neq j. \end{cases} \tag{A.10}$$

In each case, the initial values $Q_{ij}(0)$ are calculated as the solution to equation (A.4) for a given $\mathbf{V}_0$. Diagonal elements $Q_{ii}(t) \underset{t\to\infty}{\longrightarrow} \infty$, so $P_{ij}(t) \underset{t\to\infty}{\longrightarrow} 0$ for all i and j. Thus,

[28]Note that for any invertible $\mathbf{Q}_t$, $\frac{d\mathbf{Q}_t^{-1}}{dt} = -\mathbf{Q}_t^{-1}\frac{d\mathbf{Q}_t}{dt}\mathbf{Q}_t^{-1}$.

$\mathbf{V}_t \underset{t\to\infty}{\longrightarrow} \mathbf{V}^*$.

Given a solution $\mathbf{Q}_t$, equations (14), (A.4), (A.6), and (A.8) together specify the solution for $\mathbf{V}_t$ in equation (10) in the text.

A.2 Proof of Proposition 2

To prove Proposition 2, we first derive the matrix $(\mathbf{BB}')^{-1}$.

Lemma 1. *For a homogeneous family with $n \geq 2$ funds, the inverse of the correlation matrix, $(\mathbf{BB}')^{-1}$, has elements b_n on the main diagonal, and elements $\bar{b}_n$ off the main diagonal, where*

$$b_n = \frac{1+(n-2)\rho}{(1-\rho)(1+(n-1)\rho)}, \tag{A.11}$$

$$\bar{b}_n = -\frac{\rho}{(1-\rho)(1+(n-1)\rho)}. \tag{A.12}$$

Proof of Lemma 1. In the homogeneous $n-$fund family, each pair of funds has correlation of idiosyncratic returns equal to ρ. The correlation matrix is

$$\mathbf{BB}' = (1-\rho)\,\mathbf{I} + \rho\mathbf{11}'.$$

where $\mathbf{I}$ is the $n \times n$ identity matrix and $\mathbf{1}$ is a $n \times 1$ vector with elements 1. The proof is complete when we show that $\mathbf{BB}'\mathbf{P} = \mathbf{I}$, where

$$\mathbf{P} = \left(b_n - \bar{b}_n\right)\mathbf{I} + \bar{b}_n\mathbf{11}',$$

with b_n and $\overline{b}_n$ given by equations (A.11) and (A.12), respectively. We calculate

$$\begin{aligned}
\mathbf{BB'P} &= \left[(1-\rho)\,\mathbf{I}+\rho\mathbf{11'}\right]\left[\left(b_n-\overline{b}_n\right)\mathbf{I}+\overline{b}_n\mathbf{11'}\right] \\
&= (1-\rho)\left(b_n-\overline{b}_n\right)\mathbf{I}+\rho\left(b_n-\overline{b}_n\right)\mathbf{11'}+(1-\rho)\,\overline{b}_n\mathbf{11'}+\rho\mathbf{11'}\overline{b}_n\mathbf{11'} \\
&= \mathbf{I}+\left(\rho\left(b_n-\overline{b}_n\right)+(1-\rho)\,\overline{b}_n+n\rho\overline{b}_n\right)\mathbf{11'}.
\end{aligned}$$

A bit of algebra shows that the second term of the final line is zero.

Proof of Proposition 2. The matrix equation (A.1) in a homogeneous family is described by two scalar equations that identify v_n and $\overline{v}_n^2$. Each element of the main diagonal is the equation

$$\omega^2 = v_n^2 b_n + 2v_n\overline{v}_n\overline{b}_n\,(n-1) + \overline{v}_n^2\,(n-1)\left(b_n+\overline{b}_n\,(n-2)\right), \tag{A.13}$$

where b_n and $\overline{b}_n$ are given by Lemma 1. Similarly, each off-diagonal element is the equation:

$$\omega^2\lambda = v_n^2\overline{b}_n + 2v_n\overline{v}_n\left(b_n+\overline{b}_n\,(n-2)\right) + \overline{v}_n^2\left(b_n\,(n-2)+\overline{b}_n\left(n-1+(n-2)^2\right)\right). \tag{A.14}$$

To solve these equations, first substitute for b_n and $\overline{b}_n$, using equations (A.11) and (A.12), respectively. Then substitute for $\overline{v}_n$, using $\overline{v}_n = \phi_n v_n$, where ϕ_n is the correlation between the composite skills of two funds in the homogeneous family conditional on investors' information set. Set equal the ratios of the left- and right-hand sides. After some algebra, a quadratic equation identifying ϕ_n emerges:

$$0 = K_1\phi_n^2 - 2K_2\phi_n + K_3, \tag{A.15}$$

with the coefficients

$$K_1 = 1+(n-1)\,(\lambda+\rho-1) \tag{A.16}$$

$$K_2 = (n-1)\,\rho\lambda+1, \tag{A.17}$$

$$K_3 = \rho + \lambda + \lambda\rho\,(n-2)\,. \tag{A.18}$$

For the case $\rho = \frac{n-2}{n-1} - \lambda$, we find $K_1 = 0$, $\phi_n = \frac{n-2}{2(n-1)}$, and

$$v_n = \omega \frac{2}{n}\sqrt{(1-\rho)\,(n-1)\,((n-1)\rho+1)}, \tag{A.19}$$

$$\overline{v}_n = \frac{n-2}{2\,(n-1)} v_n. \tag{A.20}$$

For the case $\rho \neq \frac{n-2}{n-1} - \lambda$, the discriminant of equation (A.15), $K_2^2 - K_1 K_3$, is positive and the equation has two real roots. One root is

$$\phi_n = \rho + K_\rho \frac{K_\rho - K_\lambda}{K_1} \tag{A.21}$$

where

$$K_\tau = \sqrt{(1-\tau)\,((n-1)\,\tau+1)},\ \ \tau = \rho, \lambda. \tag{A.22}$$

Numerical evaluation of the second root gives values $|\phi_n| > 1$, which is inconsistent with the definition of a correlation. With some algebra, we then find that the variance and covariance in equations (17) and (18), respectively, solve equations (A.13) and (A.14), and that $v_n = \overline{v}_n \phi_n$, where ϕ_n is given in equation (A.21). It is evident from equation (17) that $v_n \leq \omega$. When $\rho = \lambda$, equation (A.21) gives $\phi_n = \rho = \lambda$, and equation (A.19) gives $v_n = \omega$. When $\rho \neq \lambda$, equation (A.21) gives $\phi_n \neq \rho$, and equation (A.19) gives $v_n < \omega$.

A.3 Proof of Corollary 1

(i) $n = 2$. First consider the case $\rho \neq -\lambda$. We substitute out K_ρ, K_λ, K_1 and K_2 in equations (17) and (18) using (A.22), (A.16) and (A.17). After some algebra, equation (17) becomes (19), and (18) becomes (20). For the case $\rho = -\lambda$, equations (A.19) and (A.20) yield $v_2 = \omega\sqrt{(1-\rho)(1+\rho)}$ and $\bar{v}_2 = 0$, which are consistent with equations (19) and (20).

(ii) $n \to \infty$. First consider the case $\rho \neq \frac{n-2}{n-1} - \lambda$. We again substitute out K_ρ, K_λ, K_1 and K_2 in equations (17) and (18) using (A.22), (A.16) and (A.17). We then evaluate the limits of equations (17) and (18) as $n \to \infty$. After some algebra, equation (17) becomes (21), and (18) becomes (22). For the case $\rho = \frac{n-2}{n-1} - \lambda$, equations (A.19) and (A.20) yield $v = 2\omega\sqrt{(1-\rho)\rho}$ and $\bar{v} = \omega\sqrt{(1-\rho)\rho}$, which are consistent with equations (21) and (22) as $\rho \to 1 - \lambda$.

A.4 Proof of Proposition 3

The matrix $\mathbf{S}^*$ in the homogeneous family has on- and off-diagonal elements, s_n and $\frac{\bar{s}_n}{n-1}$, respectively. Using $\mathbf{S}^* = \mathbf{V}^* (\mathbf{BB}')^{-1}$ and the solution for $(\mathbf{BB}')^{-1}$ in Lemma 1, we have

$$s_n = \left(v_n + (v_n - \overline{v}_n)\frac{(n-1)\,\rho}{1-\rho}\right)\frac{1}{1+(n-1)\,\rho}, \tag{A.23}$$

$$\overline{s}_n = \frac{\overline{v}_n - \rho v_n}{1-\rho}\frac{n-1}{1+(n-1)\,\rho}. \tag{A.24}$$

For the case defined by $\rho = \frac{n-2}{n-1} - \lambda$, substitute for v_n and $\overline{v}_n$ using equations (A.19) and (A.20). This gives

$$s_n = v_n\frac{2+(n-2)\,\rho}{2\,(1-\rho)\,(1+(n-1)\,\rho)}, \tag{A.25}$$

$$\overline{s}_n = v_n\frac{1}{1-\rho}\left(\frac{n-2-2\,(n-1)\,\rho}{2(1+(n-1)\,\rho)}\right). \tag{A.26}$$

Alternatively when $\rho \neq \frac{n-2}{n-1} - \lambda$, substitute in equations (A.23) and (A.24) using $\overline{v}_n = \phi_n v_n$, and use equations (A.21) and (A.22) to obtain:

$$s_n = v_n\left(1 - \left(1 - \frac{K_\lambda}{K_\rho}\right)\frac{(n-1)\,\rho}{K_1}\right),$$

$$\overline{s}_n = v_n\frac{n-1}{K_1}\left(1 - \frac{K_\lambda}{K_\rho}\right).$$

Finally, substitute for K_1 using equation (A.16) to obtain equations (23) and (24).

A.5 Proof of Corollary 2

(i) $n = 2$. First consider the case $\rho \neq -\lambda$. Substitute out K_ρ and K_λ in equations (23) and (24) using equation (A.22), and substitute out v_2 using (19). After some algebra, equation (23) becomes (25), and (24) becomes (26). For the case $\rho = -\lambda$, first substitute out v_n and $\bar{v}_n$ in equations (A.25) and (A.26) using (A.19) and (A.20). Equations (A.25) and (A.26) then yield $s_2 = \omega \frac{1}{\sqrt{(1-\rho)(1+\rho)}}$, and $\bar{s}_2 = \omega \frac{-\rho}{\sqrt{(1-\rho)(1+\rho)}}$. These are equivalent to (25) and (26) for $\rho = -\lambda$.

(ii) $n \to \infty$. First consider the case $\rho \neq \frac{n-2}{n-1} - \lambda$. Substitute out K_ρ and K_λ in equations (23) and (24) using equation (A.22), and substitute out v_n using (17). Evaluate the limits of equations (23) and (24) as $n \to \infty$. Note that nonsingularity of $\mathbf{BB}'$ requires $\rho \geq 0$. If $\rho > 0$, equations (27) and (28) are obtained by applying the L'Hopital's rule. If $\rho = 0$, we have

$$\lim_{n\to\infty} s_n = \omega\sqrt{(1-\lambda)}, \quad \lim_{n\to\infty} \bar{s}_n = \begin{cases} 0, & \lambda = 0, \\ \infty, & \lambda \neq 0, \end{cases}$$

which are consistent with equations (27) and (28). For the case $\rho = \frac{n-2}{n-1} - \lambda$, substitute out v_n and $\bar{v}_n$ in equations (A.25) and (A.26) using (A.19) and (A.20). Taking the limit as $n \to \infty$ yields $s = \omega \frac{\sqrt{\rho}}{\sqrt{1-\rho}}$, and $\bar{s} = \omega \frac{1-2\rho}{\sqrt{(1-\rho)\rho}}$. These results are equivalent to equations (27) and (28) as $\rho \to 1 - \lambda$.

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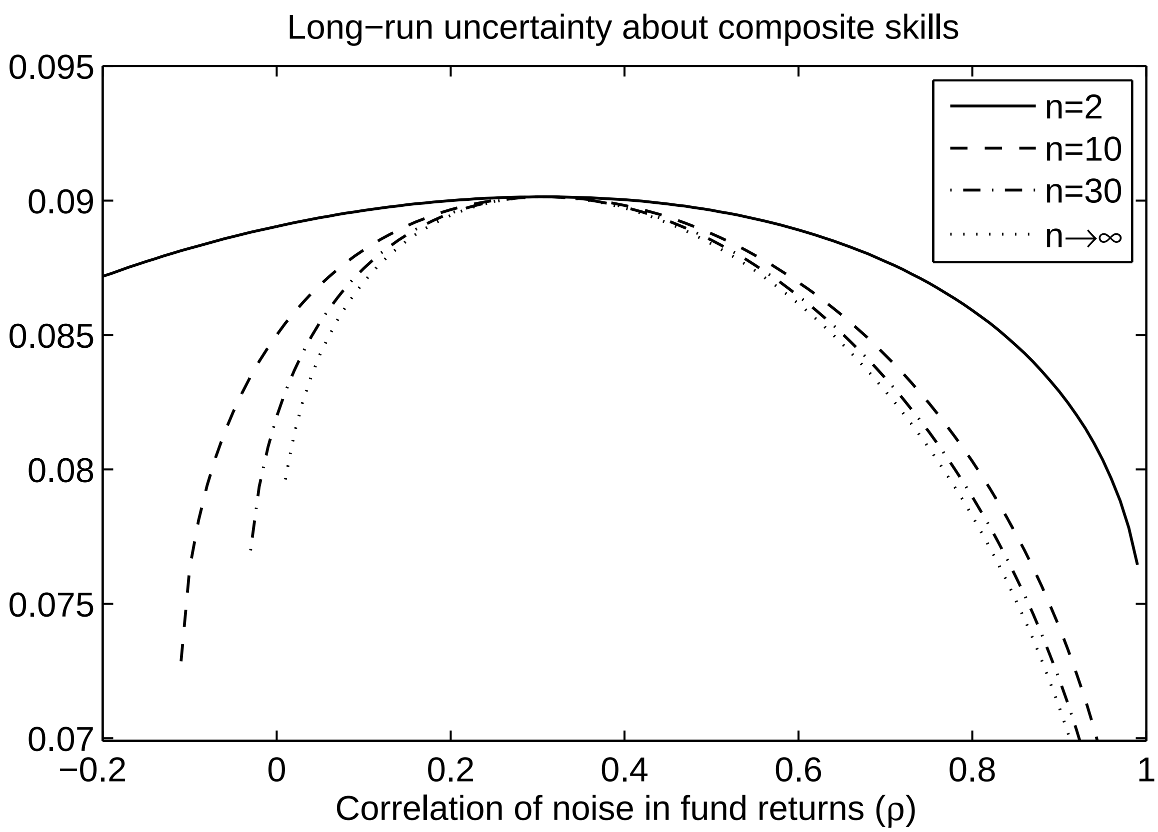

Figure 1: **Long-run uncertainty about composite skills.** This figure shows the long-run conditional variance (v_n) of a fund's skill under investors' information set. The solid line represents a family with two funds, the dashed line represents a family of 10 funds, the dot-dashed line represents a family of 30 funds, while the dotted line represents the limiting case when the number of funds in a family goes to infinity.The horizontal axis is the correlation of noise in fund returns. The other parameter values are: volatility of fund skill, ω=0.15; volatility of family skill, ω_F=0.10; weight of family skills in the composite skill, $\beta = 0.5$. These parameter values imply that the instantaneous correlation between the true skills of two funds is λ=0.31 (according to equation (7)).

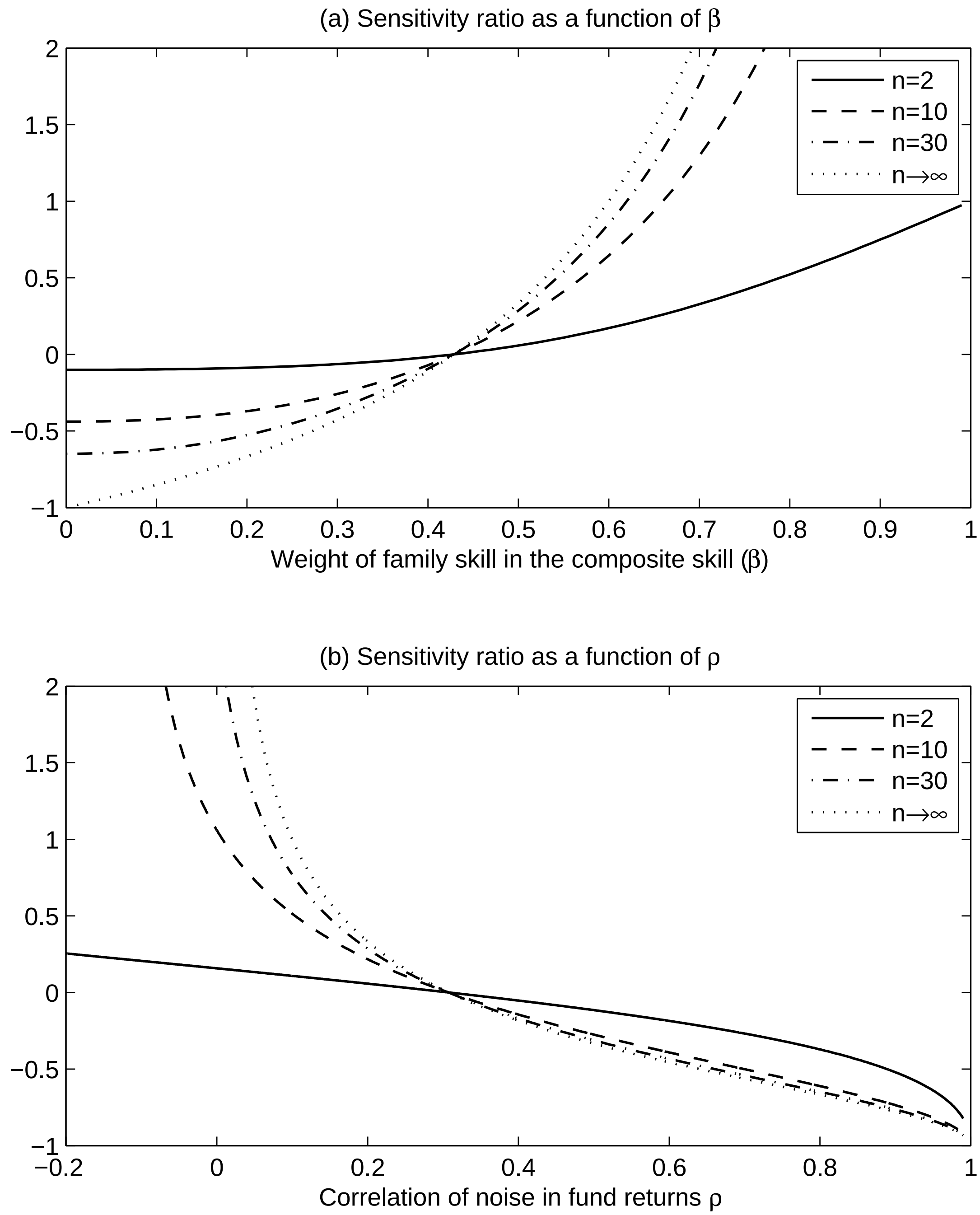

Figure 2: **Ratio of cross-sensitivity to direct sensitivity.** This figure shows the ratio $\frac{\bar{s}_n}{s_n}$ as a function of β (Panel (a)), and as a function of ρ (Panel (b)). s_n and $\bar{s}_n$ are the sensitivities of the conditional skill estimate to a fund's own unexpected performance and the average unexpected performance of other funds in the same family, respectively. The different curves correspond to different family sizes (n=2, 10, 30 and ∞, respectively). $\omega = 0.15$ and $\omega = 0.10$ in both panels, $\rho = 0.2$ in Panel (a), and $\beta = 0.5$ in Panel (b) (which implies $\lambda = 0.31$).

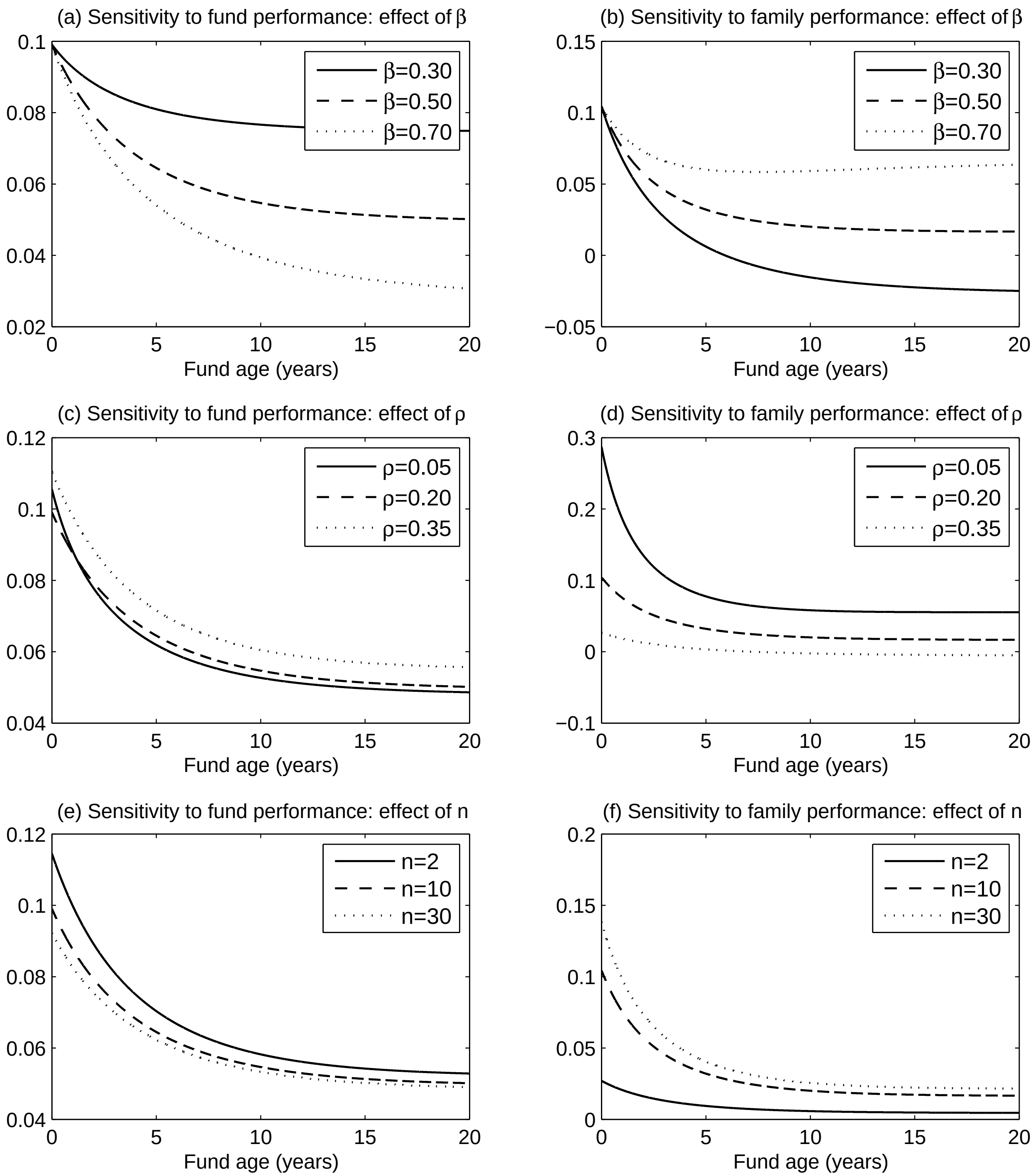

Figure 3: **Sensitivities of beliefs over time.** The left and right panels show sensitivities of the conditional skill estimate to fund and family performance, respectively. Panel (a) and (b) illustrate the effects of the weight of family skill in the composite skill (β). Panel (c) and (d) illustrate the effects of the correlation of noise in fund returns (ρ). Panel (e) and (f) illustrate the effects of family size (n). All parameters are fixed at the base case level given in Table 1, except for the one whose values are explicitly marked in each panel.

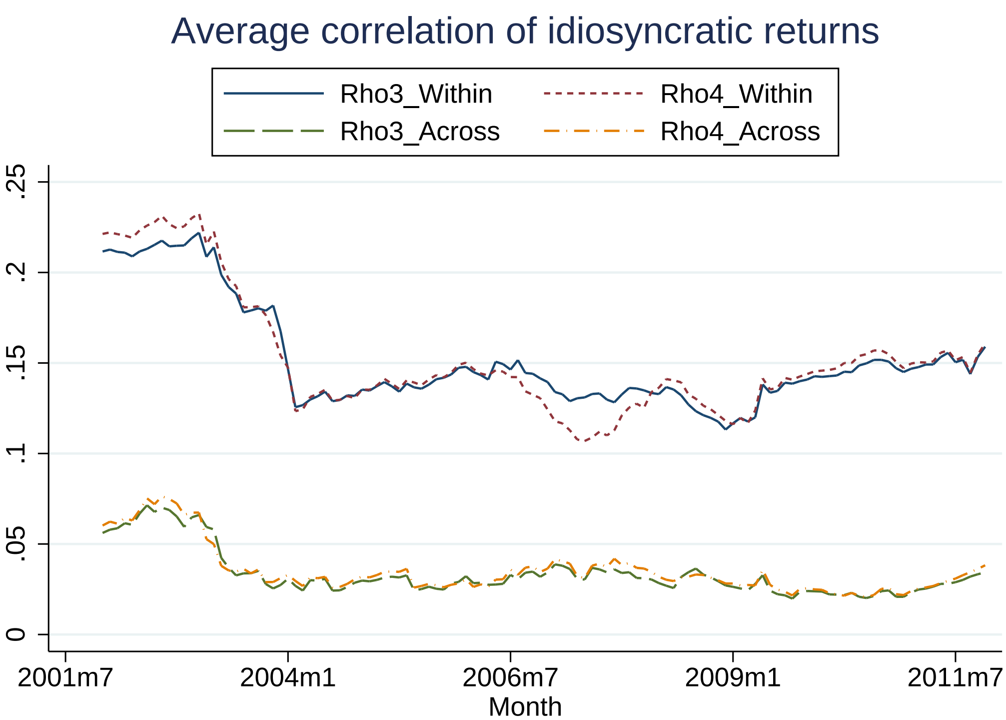

Figure 4: **Correlation of idiosyncratic returns: within vs. across families** $Rho3\_Within$ and $Rho4\_Within$ are average correlations of idiosyncratic returns within families. They are calculated by averaging all pairwise correlations of idiosyncratic returns between funds within the same family. $Rho3\_Across$ and $Rho4\_Across$ are average pairwise correlations of idiosyncratic returns between 278 standalone funds. The idiosyncratic returns are estimated either using the Fama-French three-factor model (for $Rho3\_Within$ and $Rho3\_Across$) or the Carhart four-factor model (for $Rho4\_Within$ and $Rho4\_Across$). Both idiosyncratic returns and correlations are estimated with rolling windows of 36 months.

Table 1: **Base case parameter values**

This table summarizes the base case parameter values used in generating Figure 3.

n	family size	10
ρ	correlation of noise in fund returns	0.20
β	weight of family skill in composite skill	0.5
ω	volatility of fund skill	0.15
ω_F	volatility of family skill	0.10
$v_{n,0}$	initial variance of composite skill	0.12
$\overline{v}_{n,0}$	initial covariance of composite skills	0.05
k	mean-reversion rate of skills	0.05

Table 2: **Summary statistics**

This table present summary statistics at the fund-month level. Our mutual fund sample consists of 2459 domestic equity funds from 686 fund families during the period from January 1999 to December 2011. Total net asset (TNA), return, and expense ratio are aggregated across all share classes of the same fund. Fund age is measured by the age of the oldest share class. Fund flow is calculated as asset growth rate minus fund return. Alpha, idiosyncratic volatility, and information ratio (=alpha divided by volatility of residuals) are estimated by the Fama-French three-factor model and the Carhart four factor model using rolling windows of 36 months. Correlation of idiosyncratic returns is the average pairwise correlation of the factor model residuals within a family, estimated also using rolling windows of 36 months. Manager overlap rate for a pair of funds is defined as the number of managers managing both funds divided by the average number of managers across the two funds. For a fund family, it is the average across all pairs of its member funds. Number of funds within family is the total number of funds a family is offering simultaneously. Correlations, manager overlap rate, and number of funds within family are estimated at the family level and then assigned to all member funds. Variables estimated using rolling windows of 36 months are only calculated after a fund has been affiliated with a given fund family for at least 36 months.

Variable	**Mean**	**Std. Dev.**	**N**
TNA (million dollar)	1281.745	5348.13	279829
Fund age (year)	12.801	12.994	280043
Monthly return (%)	0.401	5.814	277749
Monthly fund flow (%)	0.278	4.295	271730
Annual expense ratio (%)	1.259	0.457	275370
Alpha (3-factor, %)	-0.099	0.399	177012
Alpha (4-factor, %)	-0.103	0.384	177012
Idiosyncratic volatility (3-factor, %)	0.015	0.009	177012
Idiosyncratic volatility (4-factor, %)	0.014	0.008	177012
Information ratio (3-factor) ($Perf$)	-0.097	0.248	177012
Information ratio (4-factor) ($Perf$)	-0.103	0.257	177012
Correlation of idiosyncratic returns (3-factor) (Rho)	0.188	0.187	184244
Correlation of idiosyncratic returns (4-factor) (Rho)	0.183	0.177	184244
Manager overlap rate ($Beta$ proxy)	0.192	0.278	248054
Number of funds within family (N)	12.212	12.21	280043

Table 3: **Responses of fund flows to fund performance and family performance**

This table shows the results of Fama-MacBeth regressions of monthly fund flows (in percentage) on various explanatory variables. The coefficients are the time series averages of 120 cross-sectional regressions from January 2002 to December 2011. $Perf$ is a fund's monthly alpha estimated over the 36-month period ending at the end of the previous month, divided by the fund's monthly idiosyncratic volatility estimated over the same period; $Perf^2$ is the square of $Perf$; $FamPerf$ is the average $Perf$ of all other funds in the same family; $Log(TNA)$, $Log(Age)$, and $Log(N)$ are the natural logarithms of lagged total net asset value, fund age and number of funds in the family, respectively; *Expense* is the lagged expense ratio (in percentage); *Beta* is the lagged average pairwise manager overlap rate within a family. *Rho* is the average pairwise correlation of idiosyncratic returns estimated for each fund family over the 36-month period ending at the end of the previous month. The model also includes ten interaction terms of fund and family performance with various fund and family characteristics, including *Beta*, *Rho*, $Log(AGE)$, $Log(TNA)$, and $Log(N)$. *Beta*, *Rho*, $Log(TNA)$, $Log(Age)$, and $Log(N)$, and *Expense* are adjusted by subtracting their contemporaneous sample means. The monthly fund flow is adjusted by subtracting the contemporaneous mean of all funds in the same Lipper class. The first three columns report the results when $Perf$ and *Rho* are estimated from the three-factor model, while last three columns report results estimated using the four-factor model. The t-statistics are Newey-West corrected (with three lags) for autocorrelation in the estimated coefficients.

	three-factor model			four-factor model		
	(1)	(2)	(3)	(4)	(5)	(6)
$Perf$	4.069***	4.253***	4.042***	3.893***	4.028***	3.855***
	(14.67)	(15.09)	(14.69)	(14.24)	(14.99)	(14.35)
$Perf^2$	3.035***	3.222***	2.991***	2.711***	2.809***	2.653***
	(7.24)	(7.37)	(7.13)	(7.20)	(7.55)	(7.27)
$FamPerf$	0.662***	0.557***	0.653***	0.765***	0.634***	0.754***
	(3.95)	(3.24)	(3.92)	(4.47)	(3.72)	(4.50)
$Beta * Perf$	-1.542***	-1.567***	-1.554***	-1.387***	-1.392***	-1.384***
	(-3.92)	(-4.11)	(-4.10)	(-3.78)	(-3.84)	(-3.86)
$Beta * FamPerf$	-0.063	-0.314	-0.163	0.057	-0.210	-0.041
	(-0.12)	(-0.62)	(-0.32)	(0.12)	(-0.46)	(-0.09)
$Rho * Perf$	3.158***	3.206***	3.160***	3.205***	3.270***	3.193***
	(8.39)	(8.12)	(8.48)	(8.86)	(8.81)	(8.88)
$Rho * FamPerf$	-1.181**	-1.164*	-1.143*	-1.336**	-1.337**	-1.290**
	(-1.99)	(-1.88)	(-1.86)	(-2.29)	(-2.15)	(-2.08)
$Log(N) * Perf$	-0.418***	-0.391***	-0.440***	-0.370***	-0.356***	-0.394***
	(-6.51)	(-5.20)	(-5.60)	(-6.24)	(-5.56)	(-5.68)
$Log(N) * FamPerf$	0.602**	0.645**	0.739***	0.624***	0.610**	0.724***
	(2.37)	(2.40)	(2.65)	(2.73)	(2.42)	(2.78)
$Log(Age) * Perf$	-0.998***		-1.024***	-0.941***		-0.966***
	(-8.05)		(-8.15)	(-8.62)		(-8.14)
$Log(Age) * FamPerf$	-0.325		-0.130	-0.350*		-0.189
	(-1.43)		(-0.49)	(-1.70)		(-0.79)
$Log(TNA) * Perf$		-0.129***	0.033		-0.105***	0.042
		(-3.24)	(0.81)		(-3.10)	(1.07)
$Log(TNA) * FamPerf$		-0.208***	-0.229***		-0.175**	-0.187**
		(-2.97)	(-2.69)		(-2.51)	(-2.28)
$Beta$	-0.180	-0.208	-0.181	-0.097	-0.131	-0.092
	(-1.28)	(-1.42)	(-1.27)	(-0.67)	(-0.89)	(-0.64)
Rho	0.224*	0.213*	0.233*	0.218*	0.227*	0.217*
	(1.76)	(1.72)	(1.79)	(1.77)	(1.88)	(1.76)
$Log(N)$	-0.007	0.031	-0.011	-0.003	0.028	-0.011
	(-0.27)	(1.26)	(-0.45)	(-0.12)	(1.11)	(-0.46)
$Log(Age)$	-0.587***		-0.589***	-0.586***		-0.595***
	(-12.67)		(-12.12)	(-12.33)		(-11.79)
$Log(TNA)$		-0.099***	0.003		-0.091***	0.011
		(-8.13)	(0.24)		(-6.75)	(0.80)
$Expense$	-0.578***	-0.559***	-0.564***	-0.539***	-0.518***	-0.520***
	(-13.30)	(-11.31)	(-11.48)	(-13.48)	(-11.45)	(-11.48)
Constant	0.295***	0.302***	0.294***	0.327***	0.329***	0.325***
	(5.68)	(5.69)	(5.74)	(6.04)	(6.05)	(6.11)
Observations	152077	152077	152077	152077	152077	152077
R^2	0.060	0.053	0.060	0.057	0.050	0.058

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: **Mutual fund flow sensitivities: subsample analysis**

This table shows mutual fund flow sensitivities to fund and family performance for two opposite subsamples. Sample 1 consists of funds in families in which the common-noise effect is likely to be dominant, i.e., families with below-average manager overlap rate (after accounting for the number of funds in the family), above-average idiosyncratic return correlation, and below-average number of funds. Sample 2 consists of funds in families in which the common-skill effect is likely to be dominant, i.e., families with the opposite characteristics. The results are obtained from Fama-MacBeth regressions of monthly fund flows (in percentage) on various explanatory variables from January 2002 to December 2011. $Perf$ is a fund's monthly alpha estimated over the 36-month period ending at the end of the previous month, divided by the fund's monthly idiosyncratic volatility estimated over the same period; $Perf^2$ is the square of $Perf$; $FamPerf$ is the average $Perf$ of all other funds in the same family; $Log(TNA)$, $Log(Age)$, and $Log(N)$ are the natural logarithms of lagged total net asset value, fund age and number of funds in the family, respectively; *Expense* is the lagged expense ratio (in percentage). $Log(TNA)$, $Log(Age)$, $Log(N)$, and *Expense* are adjusted by subtracting the contemporaneous means of the full sample. The monthly fund flow is adjusted by subtracting the contemporaneous mean of all funds in the same Lipper class. Panel A reports the results when $Perf$ and *Rho* are estimated from the three-factor model, while Panel B reports results estimated using the four-factor model. The *t*-statistics are Newey-West corrected (with three lags) for autocorrelation in the estimated coefficients.

Panel A: Results from the three-factor model

	Sample 1			Sample 2		
	(1)	(2)	(3)	(4)	(5)	(6)
$Perf$	4.966***	4.866***	5.057***	3.175***	3.184***	3.617***
	(13.73)	(14.29)	(13.99)	(10.29)	(10.38)	(10.15)
$Perf^2$	3.208***	3.346***	3.758***	1.631***	1.813***	2.240***
	(3.70)	(4.30)	(3.91)	(3.90)	(4.24)	(4.79)
$FamPerf$	-0.862***	-0.638**	-1.055***	1.997***	1.563***	1.908***
	(-3.14)	(-2.33)	(-3.31)	(3.12)	(2.83)	(3.02)
$Log(Age) * Perf$		-1.259***			-0.694***	
		(-4.95)			(-4.18)	
$Log(Age) * FamPerf$		0.363			-1.126*	
		(1.01)			(-1.77)	
$Log(TNA) * Perf$			0.033			-0.249***
			(0.27)			(-3.24)
$Log(TNA) * FamPerf$			-0.612***			-0.710**
			(-4.25)			(-2.14)
$Log(Age)$		-0.554***			-0.745***	
		(-9.21)			(-8.08)	
$Log(TNA)$			-0.046**			-0.153***
			(-2.12)			(-4.66)
Expense		-0.456***	-0.323***		-0.785***	-0.827***
		(-4.20)	(-2.76)		(-11.29)	(-9.90)
Constant	0.228***	0.249***	0.245***	0.503***	0.337***	0.482***
	(3.39)	(3.62)	(3.48)	(4.73)	(3.39)	(3.79)
Observations	20122	20089	20089	41008	40982	40982
R^2	0.055	0.065	0.058	0.036	0.060	0.050

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Panel B: Results from the four-factor model

	Sample 1			Sample 2		
	(1)	(2)	(3)	(4)	(5)	(6)
$Perf$	4.807***	4.734***	4.882***	3.148***	3.142***	3.533***
	(12.08)	(12.92)	(12.41)	(10.15)	(10.04)	(10.51)
$Perf^2$	3.159***	3.366***	3.443***	1.523***	1.708***	2.049***
	(4.01)	(4.69)	(4.38)	(3.44)	(3.68)	(4.36)
$FamPerf$	-0.865***	-0.705***	-1.059***	1.845***	1.254**	1.390***
	(-3.43)	(-2.85)	(-3.48)	(3.11)	(2.56)	(2.65)
$Log(Age) * Perf$		-1.195***			-0.733***	
		(-4.12)			(-4.65)	
$Log(Age) * FamPerf$		0.389			-1.376*	
		(1.10)			(-1.89)	
$Log(TNA) * Perf$			0.100			-0.238***
			(1.17)			(-3.08)
$Log(TNA) * FamPerf$			-0.554***			-0.483
			(-3.42)			(-1.47)
$Log(Age)$		-0.555***			-0.775***	
		(-8.71)			(-7.07)	
$Log(TNA)$			-0.048**			-0.126***
			(-2.04)			(-3.70)
$Expense$		-0.481***	-0.367***		-0.751***	-0.780***
		(-4.27)	(-3.05)		(-11.72)	(-9.52)
Constant	0.291***	0.305***	0.302***	0.505***	0.346***	0.464***
	(4.33)	(4.50)	(4.50)	(5.25)	(4.06)	(4.10)
Observations	20299	20266	20266	38751	38734	38734
R^2	0.050	0.060	0.052	0.038	0.062	0.051

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$